# Lecture 13: Generative Models

## Overview

- Unsupervised Learning
- Generative Models
  - PixelRNN and PixelCNN
  - Variational Autoencoders (VAE)
  - Generative Adversarial Networks (GAN)

## Supervised vs Unsupervised Learning

### **Supervised Learning**

Data: (x, y)

x is data, y is label

**Goal**: Learn a *function* to map x -> y

**Examples**: Classification, regression, object detection, semantic segmentation, image captioning, etc.

#### **Unsupervised Learning**

Training data is cheap

**Data**: x ↓ Just data, no labels!

Holy grail: Solve unsupervised learning => understand structure

of visual world

**Goal**: Learn some underlying hidden *structure* of the data

**Examples**: Clustering, dimensionality reduction, feature learning, density estimation, etc.

## **Generative Models**

Given training data, generate new samples from same distribution







Generated samples  $\sim p_{\text{model}}(x)$ 

Want to learn  $p_{model}(x)$  similar to  $p_{data}(x)$ 

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Want to learn  $p_{model}(x)$  similar to  $p_{data}(x)$ 

Addresses density estimation, a core problem in unsupervised learning **Several flavors**:

- Explicit density estimation: explicitly define and solve for p<sub>model</sub>(x)
- Implicit density estimation: learn model that can sample from  $p_{model}(x)$  w/o explicitly defining it

## Why Generative Models?

- Realistic samples for artwork, super-resolution, colorization, etc.







- Generative models of time-series data can be used for simulation and planning (reinforcement learning applications!)
- Training generative models can also enable inference of latent representations that can be useful as general features

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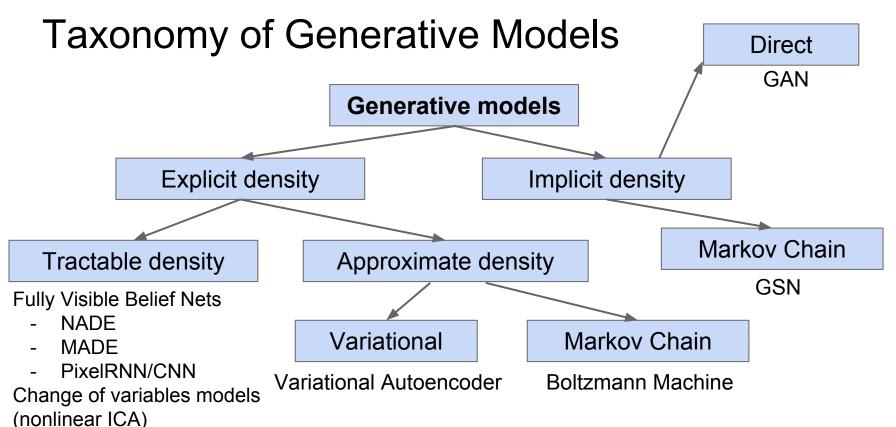
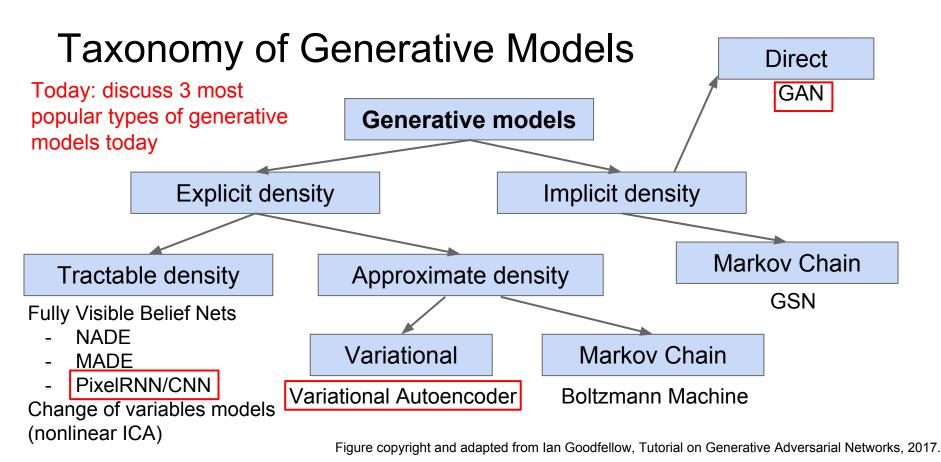


Figure copyright and adapted from Ian Goodfellow, Tutorial on Generative Adversarial Networks, 2017.



## PixelRNN and PixelCNN

## Fully visible belief network

Explicit density model

Use chain rule to decompose likelihood of an image x into product of 1-d distributions:

$$p(x) = \prod_{i=1}^n p(x_i|x_1,...,x_{i-1})$$
Likelihood of image x

Probability of i'th pixel value given all previous pixels

Then maximize likelihood of training data

## Fully visible belief network

Explicit density model

Use chain rule to decompose likelihood of an image x into product of 1-d distributions:

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Likelihood of image x

Probability of i'th pixel value given all previous pixels

Then maximize likelihood of training data

Complex distribution over pixel values => Express using a neural network!

## Fully visible belief network

Explicit density model

Use chain rule to decompose likelihood of an image x into product of 1-d distributions:

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Probability of i'th pixel value given all previous pixels

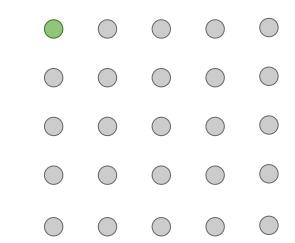
Complex distribution over pixel values => Express using a neural network!

Then maximize likelihood of training data

## PixelRNN [van der Oord et al. 2016]

Generate image pixels starting from corner

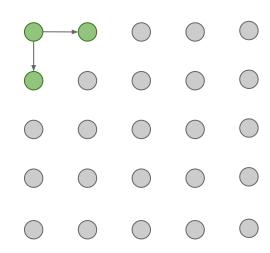
Dependency on previous pixels modeled using an RNN (LSTM)



## PixeIRNN [van der Oord et al. 2016]

Generate image pixels starting from corner

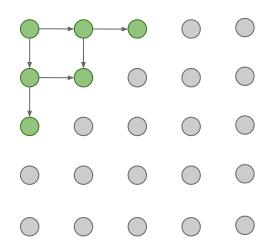
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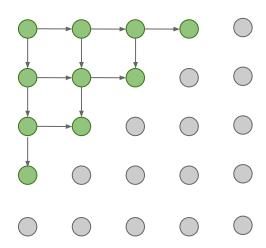


## PixeIRNN [van der Oord et al. 2016]

Generate image pixels starting from corner

Dependency on previous pixels modeled using an RNN (LSTM)

Drawback: sequential generation is slow!



## PixelCNN [van der Oord et al. 2016]

Still generate image pixels starting from corner

Dependency on previous pixels now modeled using a CNN over context region

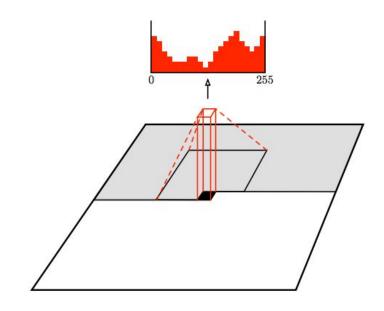


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## PixelCNN [van der Oord et al. 2016]

Still generate image pixels starting from corner

Dependency on previous pixels now modeled using a CNN over context region

Training: maximize likelihood of training images

$$p(x) = \prod_{i=1}^{n} p(x_i|x_1, ..., x_{i-1})$$

#### Softmax loss at each pixel

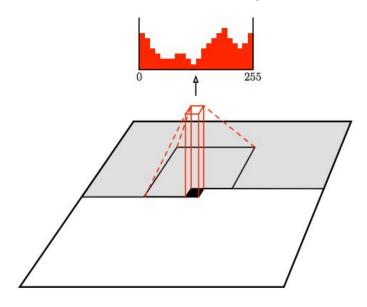


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## PixelCNN [van der Oord et al. 2016]

Still generate image pixels starting from corner

Dependency on previous pixels now modeled using a CNN over context region

Training is faster than PixelRNN (can parallelize convolutions since context region values known from training images)

Generation must still proceed sequentially => still slow

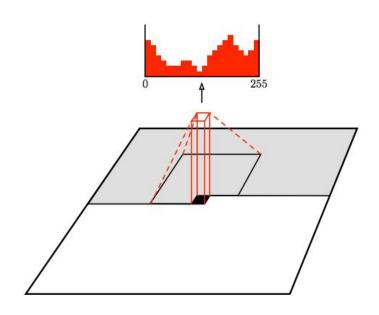


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## **Generation Samples**



32x32 CIFAR-10



32x32 ImageNet

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## PixelRNN and PixelCNN

#### Pros:

- Can explicitly compute likelihood p(x)
- Explicit likelihood of training data gives good evaluation metric
- Good samples

#### Con:

Sequential generation => slow

#### Improving PixelCNN performance

- Gated convolutional layers
- Short-cut connections
- Discretized logistic loss
- Multi-scale
- Training tricks
- Etc...

#### See

- Van der Oord et al. NIPS 2016
- Salimans et al. 2017 (PixelCNN++)

# Variational Autoencoders (VAE)

## So far...

PixelCNNs define tractable density function, optimize likelihood of training data:

$$p_{\theta}(x) = \prod_{i=1} p_{\theta}(x_i|x_1, ..., x_{i-1})$$

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PixelCNNs define tractable density function, optimize likelihood of training data:

$$p_{\theta}(x) = \prod_{i=1} p_{\theta}(x_i|x_1, ..., x_{i-1})$$

VAEs define intractable density function with latent **z**:

$$p_{ heta}(x) = \int p_{ heta}(z) p_{ heta}(x|z) dz$$

Cannot optimize directly, derive and optimize lower bound on likelihood instead

Sample from true conditional  $p_{\theta^*}(x \mid z^{(i)})$  Decoder network Sample from true prior  $p_{\theta^*}(z)$ 

We want to estimate the true parameters  $\theta^*$  of this generative model.

How to train the model?

Remember strategy for training generative models from FVBNs. Learn model parameters to maximize likelihood of training data

$$p_{\theta}(x) = \int p_{\theta}(z) p_{\theta}(x|z) dz$$

Q: What is the problem with this?

Intractable!

Data likelihood:  $p_{\theta}(x) = \int p_{\theta}(z) p_{\theta}(x|z) dz$ 

Simple Gaussian prior

Data likelihood: 
$$p_{\theta}(x) = \int p_{\theta}(z) p_{\theta}(x|z) dz$$

Data likelihood: 
$$p_{ heta}(x) = \int p_{ heta}(z) p_{ heta}(x|z) dz$$

Decoder neural network

Data likelihood: 
$$p_{\theta}(x) = \int p_{\theta}(z) p_{\theta}(x|z) dz$$

Intractible to compute p(x|z) for every z!

Data likelihood:  $p_{\theta}(x) = \int p_{\theta}(z) p_{\theta}(x|z) dz$ 

Posterior density also intractable:  $p_{\theta}(z|x) = p_{\theta}(x|z)p_{\theta}(z)/p_{\theta}(x)$ 

Data likelihood: 
$$p_{ heta}(x) = \int p_{ heta}(z) p_{ heta}(x|z) dz$$

Posterior density also intractable: 
$$p_{ heta}(z|x) = p_{ heta}(x|z)p_{ heta}(z)/p_{ heta}(x)$$

Intractable data likelihood

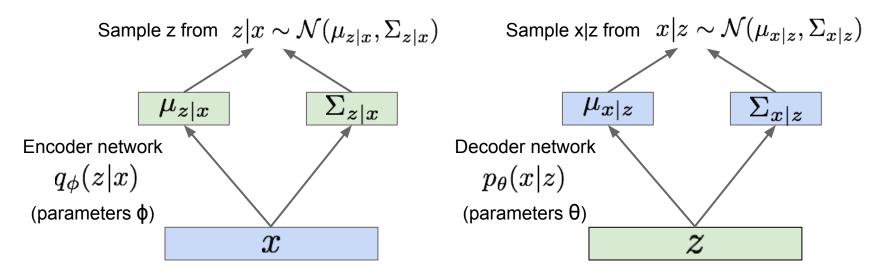
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Posterior density also intractable:  $p_{ heta}(z|x) = p_{ heta}(x|z)p_{ heta}(z)/p_{ heta}(x)$ 

Solution: In addition to decoder network modeling  $p_{\theta}(x|z)$ , define additional encoder network  $q_{b}(z|x)$  that approximates  $p_{\theta}(z|x)$ 

Will see that this allows us to derive a lower bound on the data likelihood that is tractable, which we can optimize

Since we're modeling probabilistic generation of data, encoder and decoder networks are probabilistic



Encoder and decoder networks also called "recognition"/"inference" and "generation" networks

Now equipped with our encoder and decoder networks, let's work out the (log) data likelihood:

$$\log p_{\theta}(x^{(i)}) = \mathbf{E}_{z \sim q_{\phi}(z|x^{(i)})} \left[ \log p_{\theta}(x^{(i)}) \right] \qquad (p_{\theta}(x^{(i)}) \text{ Does not depend on } z)$$

$$= \mathbf{E}_{z} \left[ \log \frac{p_{\theta}(x^{(i)} \mid z) p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \right] \qquad (\text{Bayes' Rule})$$

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$$= \mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z)} \right] + \mathbf{E}_{z} \left[ \log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z \mid x^{(i)})} \right] \qquad (\text{Logarithms})$$

$$= \mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - D_{KL}(q_{\phi}(z \mid x^{(i)}) || p_{\theta}(z)) + D_{KL}(q_{\phi}(z \mid x^{(i)}) || p_{\theta}(z \mid x^{(i)}))$$

Decoder network gives  $p_{\theta}(x|z)$ , can compute estimate of this term through sampling. (Sampling differentiable through reparam. trick, see paper.)

This KL term (between Gaussians for encoder and z prior) has nice closed-form solution!

 $p_{\theta}(z|x)$  intractable (saw earlier), can't compute this KL term :( But we know KL divergence always >= 0.

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**Tractable lower bound** which we can take gradient of and optimize! ( $p_{\theta}(x|z)$  differentiable, KL term differentiable)

Now equipped with our encoder and decoder networks, let's work out the (log) data likelihood:

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$$\geq 0$$

$$\log p_{\theta}(x^{(i)}) \geq \mathcal{L}(x^{(i)}, \theta, \phi)$$

$$\theta^{*}, \phi^{*} = \arg \max_{\theta, \phi} \sum_{i=1}^{N} \mathcal{L}(x^{(i)}, \theta, \phi)$$

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Variational lower bound ("ELBO")

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Training: Maximize lower bound

Now equipped with our encoder and decoder networks, let's work out the (log) data likelihood:

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$$= \mathbf{E}_{z} \left[\log \frac{p_{\theta}(x^{(i)} \mid z)p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})}\right] \quad (\text{Bayes' Rule}) \qquad \text{Make approximate}$$

$$\text{Reconstruct}$$

$$\text{the input data} = \mathbf{E}_{z} \left[\log \frac{p_{\theta}(x^{(i)} \mid z)p_{\theta}(z)}{p_{\theta}(z \mid x^{(i)})} \frac{q_{\phi}(z \mid x^{(i)})}{q_{\phi}(z \mid x^{(i)})}\right] \quad (\text{Multiply by constant}) \quad \text{close to prior}$$

$$= \mathbf{E}_{z} \left[\log p_{\theta}(x^{(i)} \mid z)\right] - \mathbf{E}_{z} \left[\log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z)}\right] + \mathbf{E}_{z} \left[\log \frac{q_{\phi}(z \mid x^{(i)})}{p_{\theta}(z \mid x^{(i)})}\right] \quad (\text{Logarithms})$$

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$$\geq 0$$

$$\log p_{\theta}(x^{(i)}) \geq \mathcal{L}(x^{(i)}, \theta, \phi)$$

$$\text{Variational lower bound ("ELBO")}$$

$$\underbrace{\mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - D_{KL}(q_{\phi}(z \mid x^{(i)}) \mid\mid p_{\theta}(z))}_{\mathcal{L}(x^{(i)}, \theta, \phi)}$$

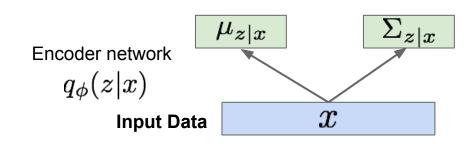
Putting it all together: maximizing the likelihood lower bound

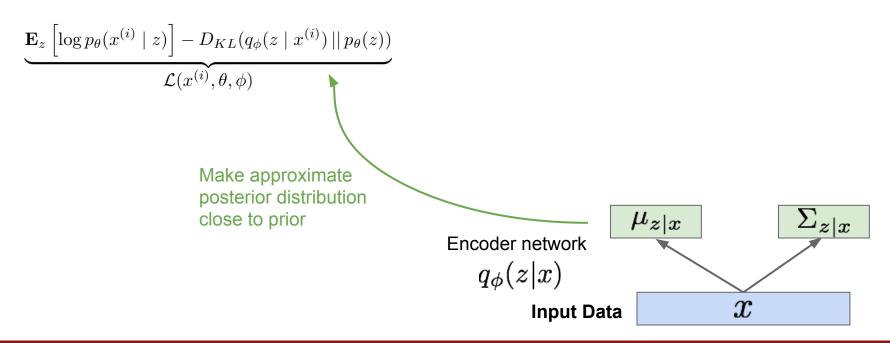
$$\underbrace{\mathbf{E}_{z} \left[ \log p_{\theta}(x^{(i)} \mid z) \right] - D_{KL}(q_{\phi}(z \mid x^{(i)}) \mid\mid p_{\theta}(z))}_{\mathcal{L}(x^{(i)}, \theta, \phi)}$$

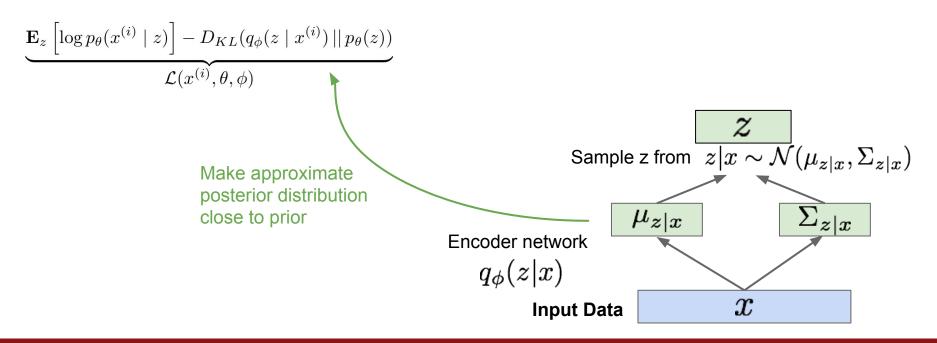
Let's look at computing the bound (forward pass) for a given minibatch of input data

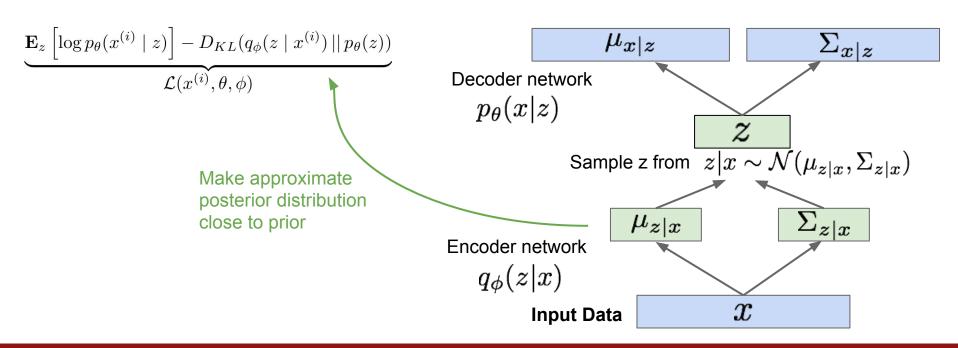
> x**Input Data**

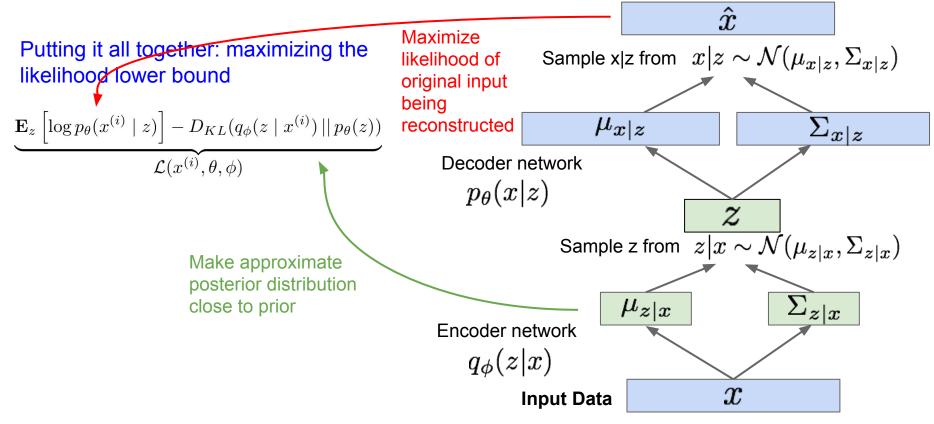
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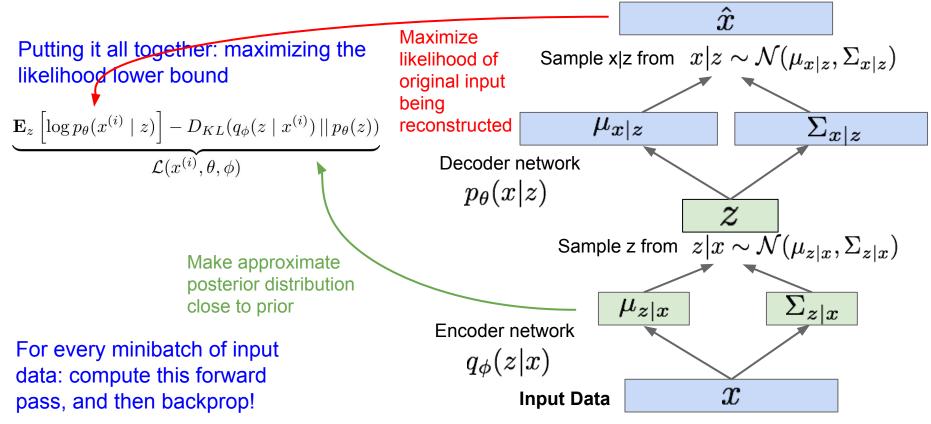




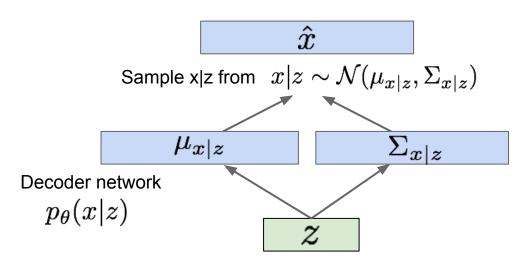








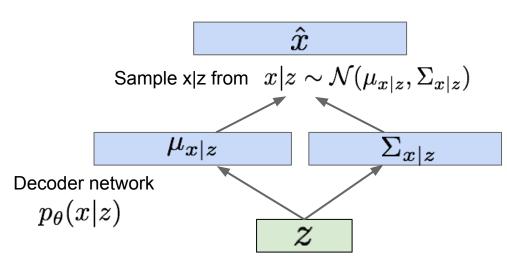
Use decoder network. Now sample z from prior!



Sample z from  $\,z \sim \mathcal{N}(0,I)\,$ 

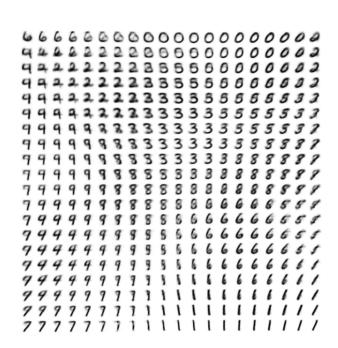
Kingma and Welling, "Auto-Encoding Variational Bayes", ICLR 2014

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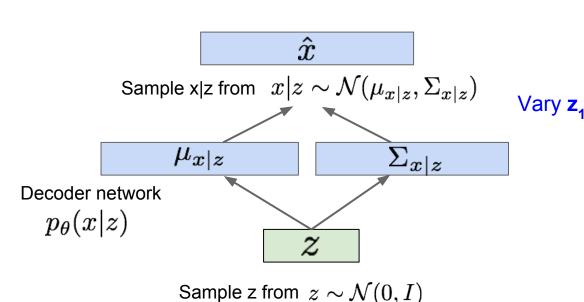


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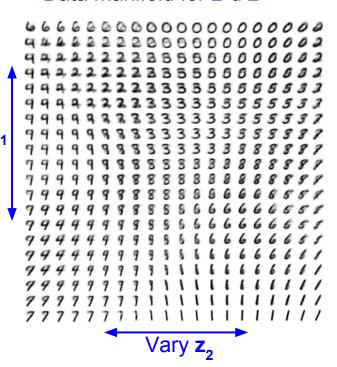
Use decoder network. Now sample z from prior!



Sample 2 noin  $z \sim \mathcal{W}(0, T)$ 

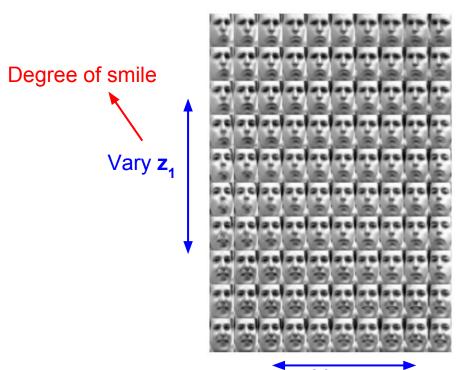
Kingma and Welling, "Auto-Encoding Variational Bayes", ICLR 2014

#### Data manifold for 2-d z



Diagonal prior on **z** => independent latent variables

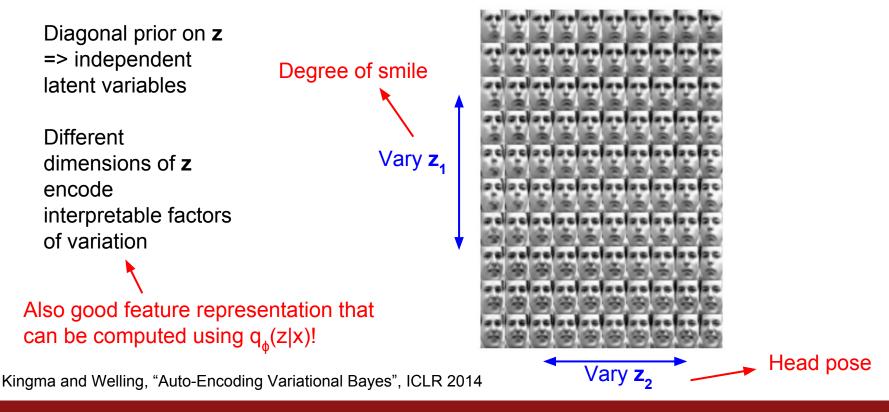
Different dimensions of **z** encode interpretable factors of variation



Kingma and Welling, "Auto-Encoding Variational Bayes", ICLR 2014

Vary **z**<sub>2</sub>

Head pose



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32x32 CIFAR-10



Labeled Faces in the Wild

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Probabilistic spin to traditional autoencoders => allows generating data

Defines an intractable density => derive and optimize a (variational) lower bound

#### **Pros**:

- Principled approach to generative models
- Allows inference of q(z|x), can be useful feature representation for other tasks

#### Cons:

- Maximizes lower bound of likelihood: okay, but not as good evaluation as PixelRNN/PixelCNN
- Samples blurrier and lower quality compared to state-of-the-art (GANs)

#### Active areas of research:

- More flexible approximations, e.g. richer approximate posterior instead of diagonal Gaussian
- Incorporating structure in latent variables

# Generative Adversarial Networks (GAN)

#### So far...

PixelCNNs define tractable density function, optimize likelihood of training data:

$$p_{\theta}(x) = \prod_{i=1}^{n} p_{\theta}(x_i|x_1, ..., x_{i-1})$$

VAEs define intractable density function with latent **z**:

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Cannot optimize directly, derive and optimize lower bound on likelihood instead

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What if we give up on explicitly modeling density, and just want ability to sample?

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Cannot optimize directly, derive and optimize lower bound on likelihood instead

What if we give up on explicitly modeling density, and just want ability to sample?

GANs: don't work with any explicit density function! Instead, take game-theoretic approach: learn to generate from training distribution through 2-player game

#### **Generative Adversarial Networks**

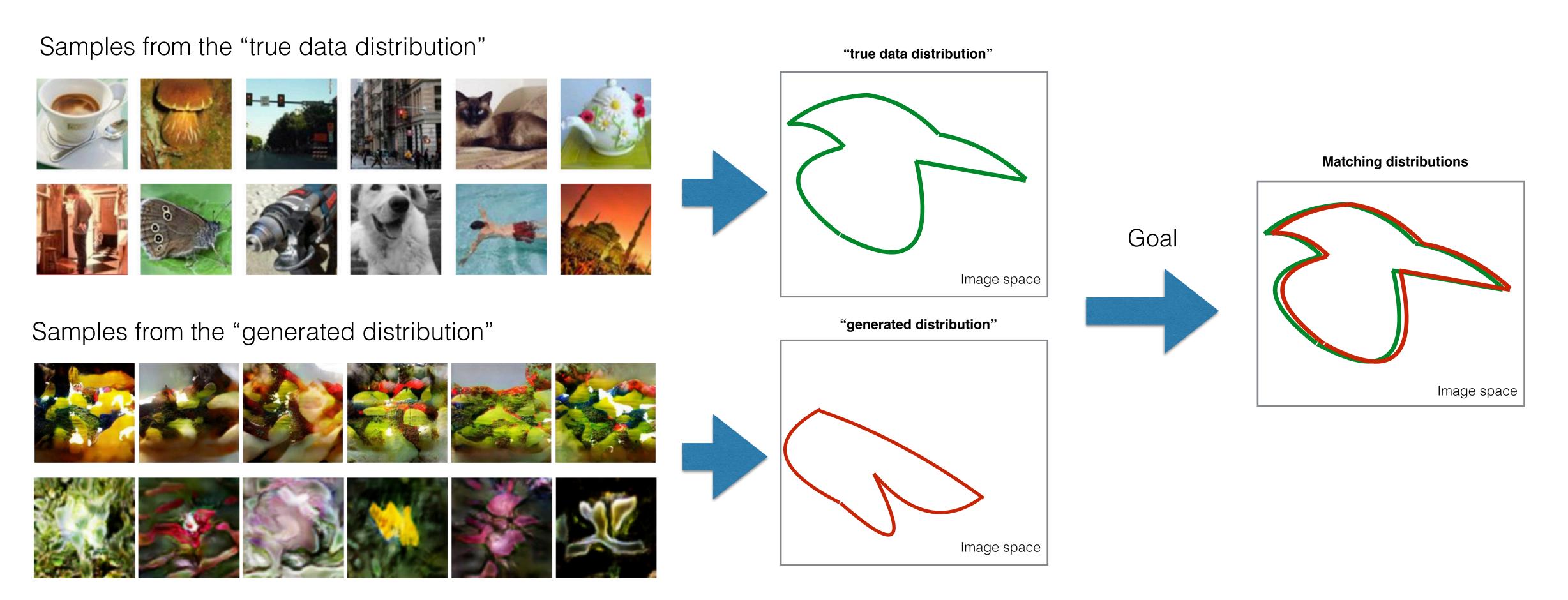
Problem: Want to sample from complex, high-dimensional training distribution. No direct way to do this!

Solution: Sample from a simple distribution, e.g. random noise. Learn transformation to training distribution.

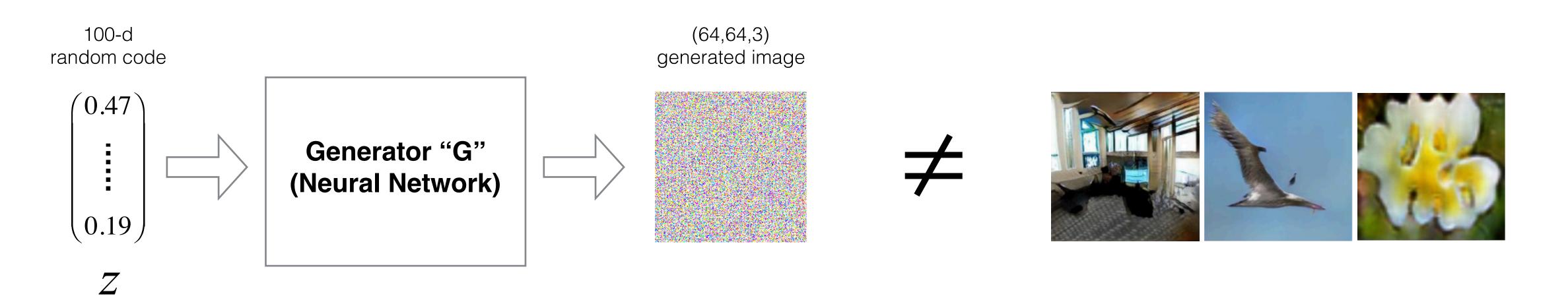
Q: What can we use to represent this complex transformation?

# **II.A - Motivation**

# **Probability distributions:**



# II.B - G/D Game



How can we train G to generate images from the true data distributions?

#### **Generative Adversarial Networks**

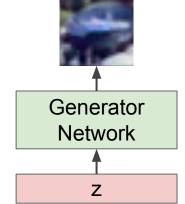
Problem: Want to sample from complex, high-dimensional training distribution. No direct way to do this!

Solution: Sample from a simple distribution, e.g. random noise. Learn transformation to training distribution.

Q: What can we use to represent this complex transformation?

A: A neural network!

Output: Sample from training distribution



Input: Random noise

# Training GANs: Two-player game

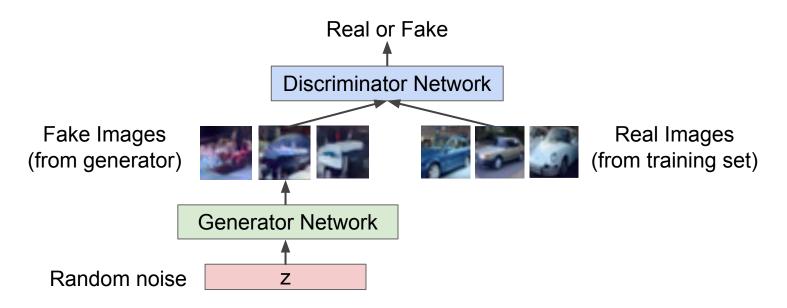
lan Goodfellow et al., "Generative Adversarial Nets", NIPS 2014

**Generator network**: try to fool the discriminator by generating real-looking images **Discriminator network**: try to distinguish between real and fake images

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# Training GANs: Two-player game

**Generator network**: try to fool the discriminator by generating real-looking images **Discriminator network**: try to distinguish between real and fake images



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# II.B - G/D Game

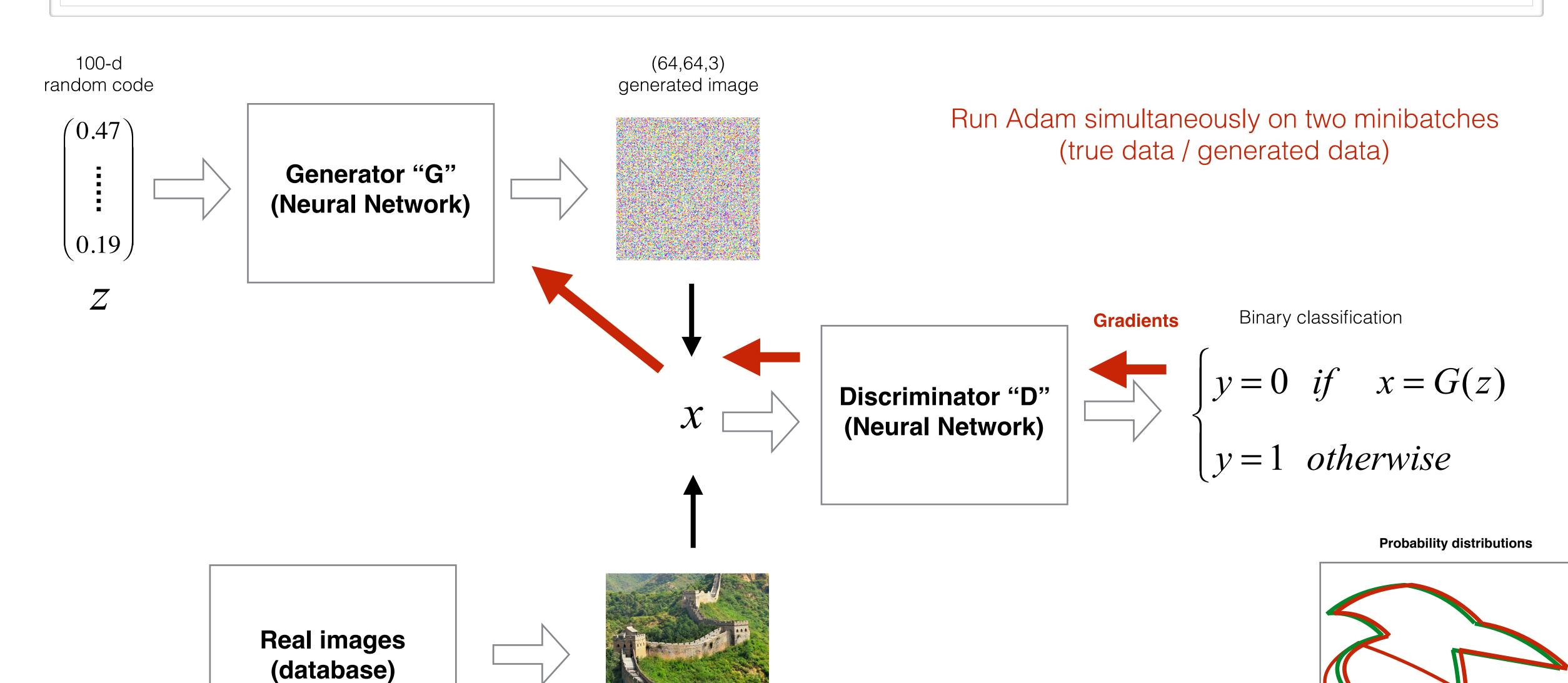


Image space

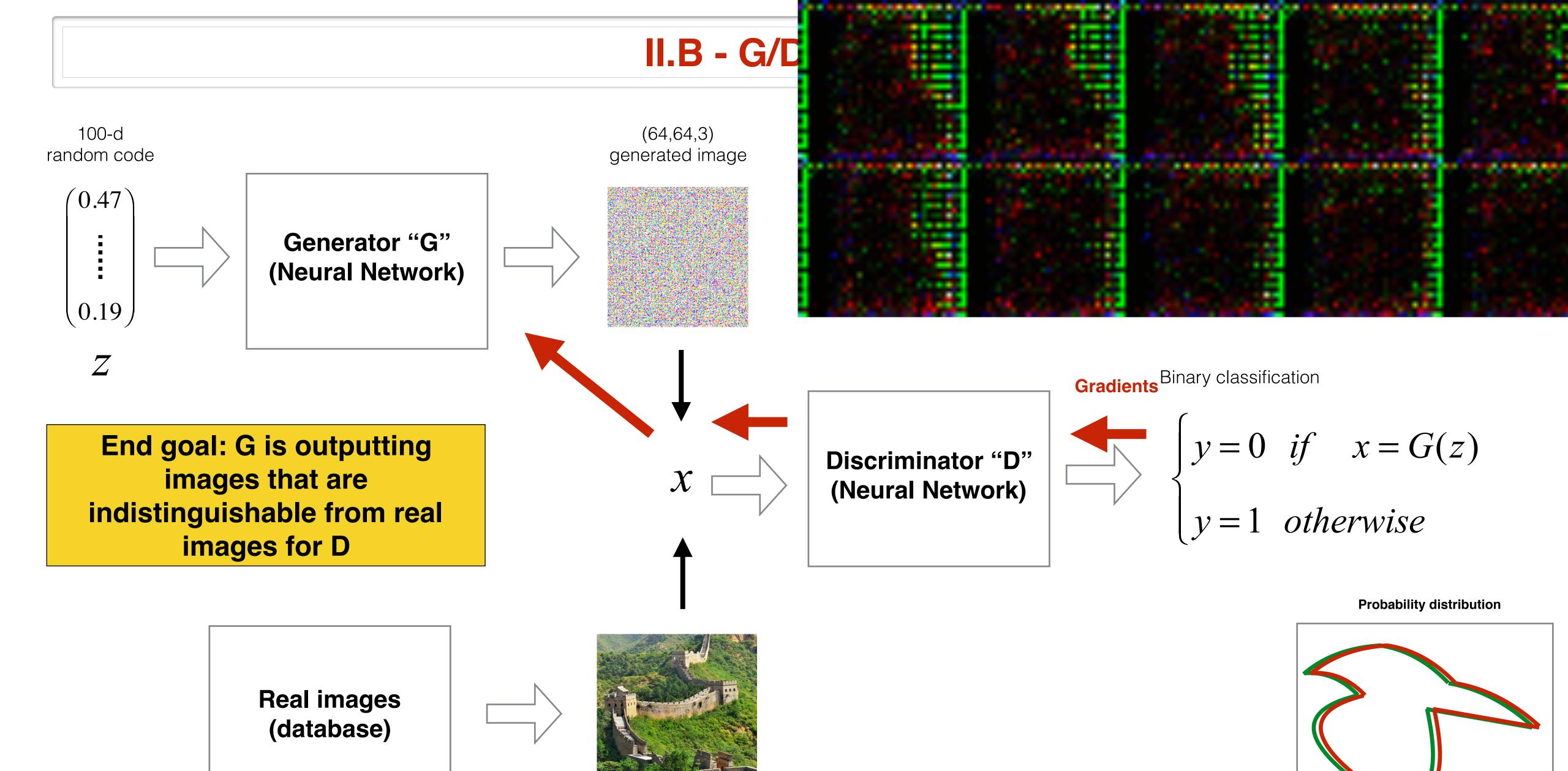


Image space

# Training GANs: Two-player game

**Generator network**: try to fool the discriminator by generating real-looking images **Discriminator network**: try to distinguish between real and fake images

Train jointly in minimax game

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log (1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

**Generator network**: try to fool the discriminator by generating real-looking images **Discriminator network**: try to distinguish between real and fake images

Train jointly in minimax game

Discriminator outputs likelihood in (0,1) of real image

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log (1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$
Discriminator output for for real data x
$$\min_{\theta_g} \max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log (1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$
Discriminator output for generated fake data G(z)

# Training GANs: Two-player game

**Generator network**: try to fool the discriminator by generating real-looking images **Discriminator network**: try to distinguish between real and fake images

Train jointly in minimax game

Discriminator outputs likelihood in (0,1) of real image

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log (1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$
Discriminator output for for real data x
$$\min_{\theta_d} \max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log (1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$
Discriminator output for generated fake data G(z)

- Discriminator  $(\theta_d)$  wants to **maximize objective** such that D(x) is close to 1 (real) and D(G(z)) is close to 0 (fake)
- Generator  $(\theta_g)$  wants to **minimize objective** such that D(G(z)) is close to 1 (discriminator is fooled into thinking generated G(z) is real)

Ian Goodfellow et al., "Generative Adversarial Nets", NIPS 2014

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log (1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

#### Alternate between:

**Gradient ascent** on discriminator

$$\max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

**Gradient descent** on generator

$$\min_{\theta_g} \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z)))$$

lan Goodfellow et al., "Generative Adversarial Nets", NIPS 2014

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log (1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

#### Alternate between:

Gradient ascent on discriminator

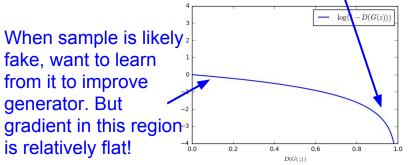
$$\max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

2. Gradient descent on generator

$$\min_{\theta_g} \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z)))$$

In practice, optimizing this generator objective does not work well!

Gradient signal dominated by region where sample is already good



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## Training GANs: Two-player game

Minimax objective function:

$$\min_{\theta_a} \max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log (1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

#### Alternate between:

Gradient ascent on discriminator

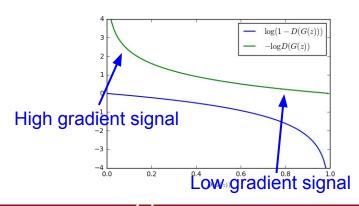
$$\max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log (1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

2. Instead: Gradient ascent on generator, different objective

$$\max_{\theta_g} \mathbb{E}_{z \sim p(z)} \log(D_{\theta_d}(G_{\theta_g}(z)))$$

Instead of minimizing likelihood of discriminator being correct, now maximize likelihood of discriminator being wrong.

Same objective of fooling discriminator, but now higher gradient signal for bad samples => works much better! Standard in practice.



# Training GANs: Two-player game

lan Goodfellow et al., "Generative Adversarial Nets", NIPS 2014

Minimax objective function:

$$\min_{\theta_a} \max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log (1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

#### Alternate between:

Gradient ascent on discriminator

$$\max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log (1 - D_{\theta_d}(G_{\theta_g}(z))) \right] \\ \underset{\text{is an active area of}}{\text{landscapes helps training,}}$$

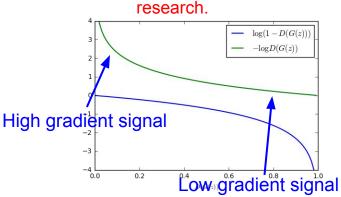
2. Instead: Gradient ascent on generator, different objective

$$\max_{\theta_g} \mathbb{E}_{z \sim p(z)} \log(D_{\theta_d}(G_{\theta_g}(z)))$$

Instead of minimizing likelihood of discriminator being correct, now maximize likelihood of discriminator being wrong.

Same objective of fooling discriminator, but now higher gradient signal for bad samples => works much better! Standard in practice.

Aside: Jointly training two networks is challenging, can be unstable. Choosing objectives with better loss landscapes helps training, is an active area of



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Ian Goodfellow et al., "Generative Adversarial Nets", NIPS 2014

#### Putting it together: GAN training algorithm

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples  $\{z^{(1)}, \ldots, z^{(m)}\}$  from noise prior  $p_q(z)$ .
- ullet Sample minibatch of m examples  $\{oldsymbol{x}^{(1)},\ldots,oldsymbol{x}^{(m)}\}$  from data generating distribution  $p_{\text{data}}(\boldsymbol{x})$ .
- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[ \log D_{\theta_d}(x^{(i)}) + \log(1 - D_{\theta_d}(G_{\theta_g}(z^{(i)}))) \right]$$

#### end for

- Sample minibatch of m noise samples  $\{z^{(1)}, \dots, z^{(m)}\}$  from noise prior  $p_q(z)$ .
- Update the generator by ascending its stochastic gradient (improved objective):

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log(D_{\theta_d}(G_{\theta_g}(z^{(i)})))$$

end for

### Putting it together: GAN training algorithm

for number of training iterations do

#### for k steps do

- Sample minibatch of m noise samples  $\{z^{(1)}, \ldots, z^{(m)}\}$  from noise prior  $p_g(z)$ .
- Sample minibatch of m examples  $\{x^{(1)}, \dots, x^{(m)}\}$  from data generating distribution  $p_{\text{data}}(x)$ .
- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[ \log D_{\theta_d}(x^{(i)}) + \log(1 - D_{\theta_d}(G_{\theta_g}(z^{(i)}))) \right]$$

Recent work (e.g. Wasserstein GAN) alleviates this problem, better stability!

Some find k=1

others use k > 1,

more stable,

no best rule.

#### end for

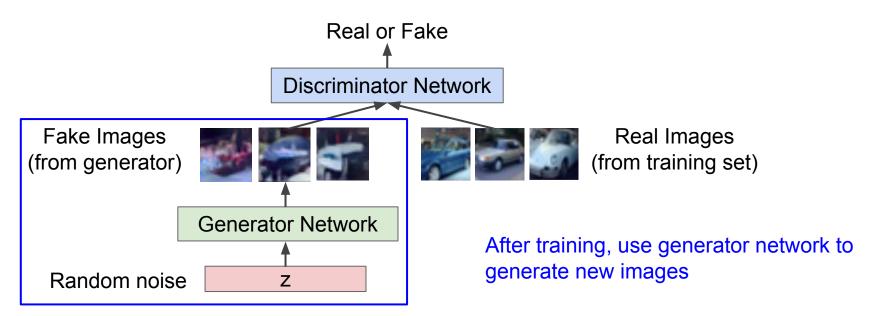
- Sample minibatch of m noise samples  $\{z^{(1)}, \dots, z^{(m)}\}$  from noise prior  $p_q(z)$ .
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end for

# Training GANs: Two-player game

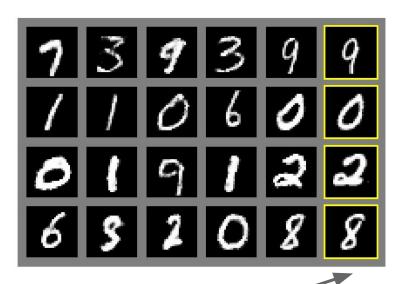
**Generator network**: try to fool the discriminator by generating real-looking images **Discriminator network**: try to distinguish between real and fake images

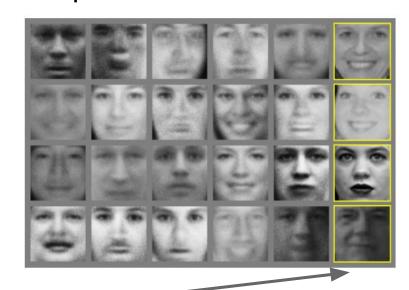


Fake and real images copyright Emily Denton et al. 2015. Reproduced with permission.

## **Generative Adversarial Nets**

### Generated samples





Nearest neighbor from training set

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## **Generative Adversarial Nets**

## Generated samples (CIFAR-10)





Nearest neighbor from training set

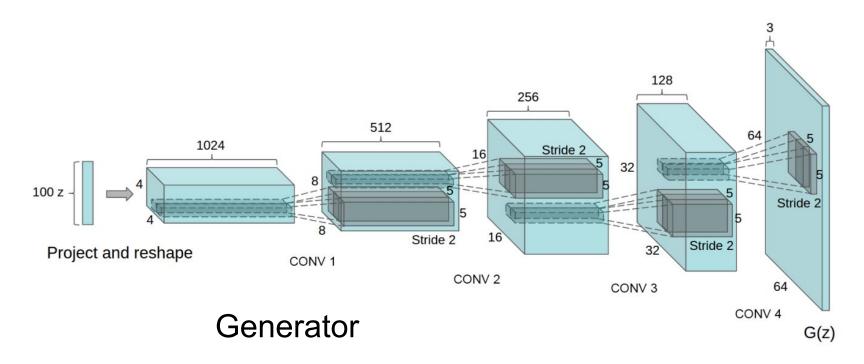
Figures copyright Ian Goodfellow et al., 2014. Reproduced with permission.

Generator is an upsampling network with fractionally-strided convolutions Discriminator is a convolutional network

Architecture guidelines for stable Deep Convolutional GANs

- Replace any pooling layers with strided convolutions (discriminator) and fractional-strided convolutions (generator).
- Use batchnorm in both the generator and the discriminator.
- Remove fully connected hidden layers for deeper architectures.
- Use ReLU activation in generator for all layers except for the output, which uses Tanh.
- Use LeakyReLU activation in the discriminator for all layers.

Radford et al, "Unsupervised Representation Learning with Deep Convolutional Generative Adversarial Networks", ICLR 2016



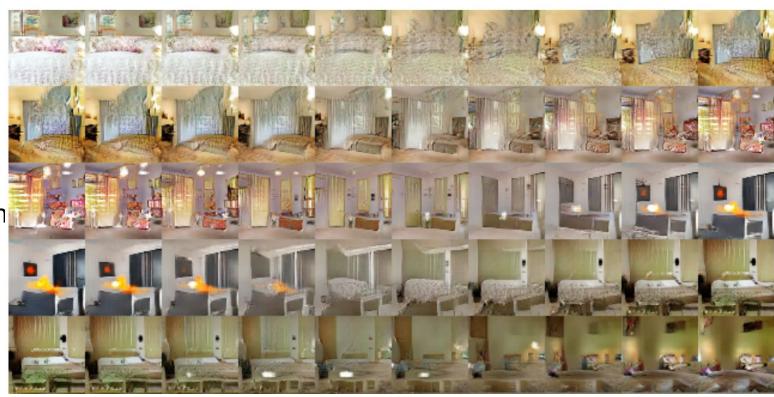
Radford et al, "Unsupervised Representation Learning with Deep Convolutional Generative Adversarial Networks", ICLR 2016

Samples from the model look amazing!



Radford et al, ICLR 2016

Interpolating between random points in laten space

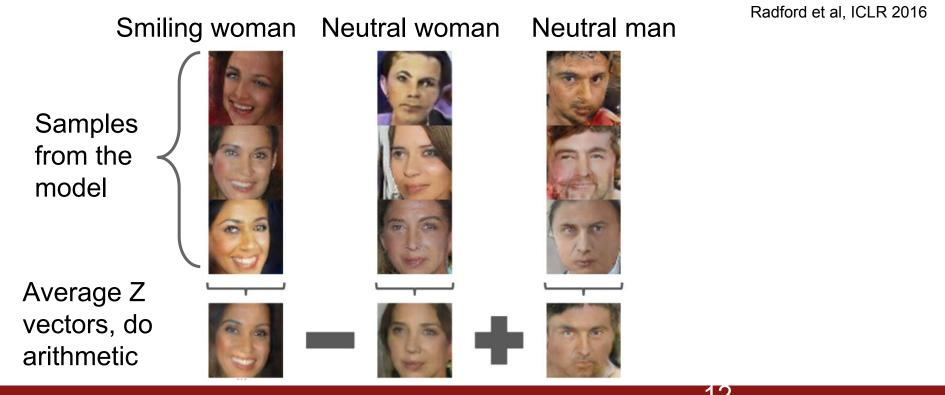


Radford et al, ICLR 2016

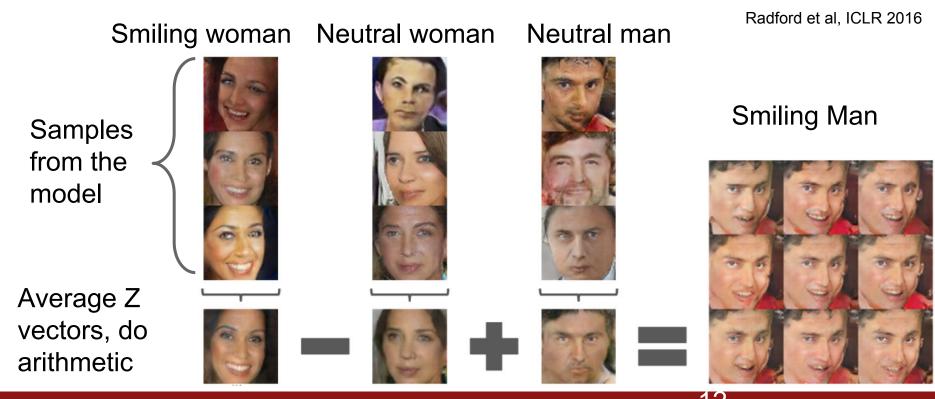
Samples from the model

Smiling woman Neutral woman Neutral man Ne

Radford et al, ICLR 2016



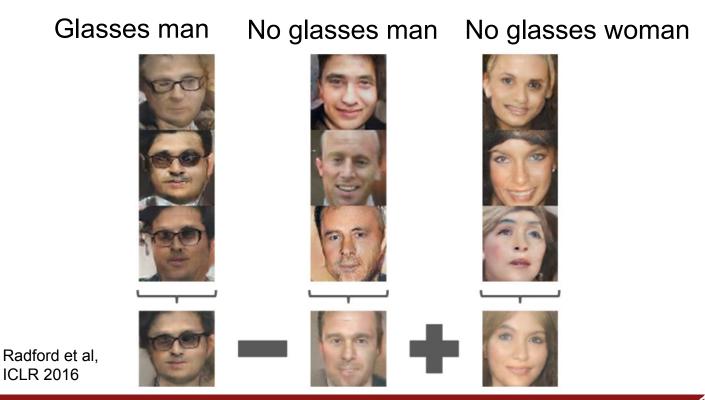
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May 18, 2017



**ICLR 2016** 



### 2017: Year of the GAN

#### Better training and generation



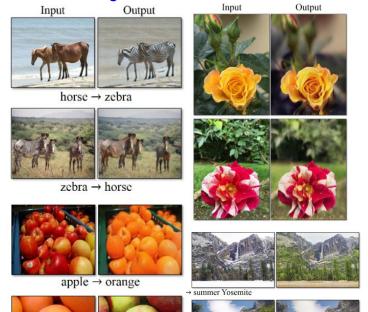


(d) Conference room LSGAN, Mao et al. 2017.



BEGAN. Bertholet et al. 2017.

#### Source->Target domain transfer



CycleGAN. Zhu et al. 2017.

### Text -> Image Synthesis

this small bird has a pink breast and crown, and black almost all black with a red primaries and secondaries.

this magnificent fellow is crest, and white cheek patch.





Reed et al. 2017.

#### Many GAN applications



Pix2pix. Isola 2017. Many examples at https://phillipi.github.io/pix2pix/

## "The GAN Zoo"

- GAN Generative Adversarial Networks
- · 3D-GAN Learning a Probabilistic Latent Space of Object Shapes via 3D Generative-Adversarial Modeling
- · acGAN Face Aging With Conditional Generative Adversarial Networks
- AC-GAN Conditional Image Synthesis With Auxiliary Classifier GANs
- AdaGAN AdaGAN: Boosting Generative Models
- AEGAN Learning Inverse Mapping by Autoencoder based Generative Adversarial Nets
- AffGAN Amortised MAP Inference for Image Super-resolution
- · AL-CGAN Learning to Generate Images of Outdoor Scenes from Attributes and Semantic Layouts
- · ALI Adversarially Learned Inference
- · AM-GAN Generative Adversarial Nets with Labeled Data by Activation Maximization
- AnoGAN Unsupervised Anomaly Detection with Generative Adversarial Networks to Guide Marker Discovery
- ArtGAN ArtGAN: Artwork Synthesis with Conditional Categorial GANs
- b-GAN b-GAN: Unified Framework of Generative Adversarial Networks
- · Bayesian GAN Deep and Hierarchical Implicit Models
- . BEGAN BEGAN: Boundary Equilibrium Generative Adversarial Networks
- . BiGAN Adversarial Feature Learning
- . BS-GAN Boundary-Seeking Generative Adversarial Networks
- CGAN Conditional Generative Adversarial Nets
- CaloGAN CaloGAN: Simulating 3D High Energy Particle Showers in Multi-Layer Electromagnetic Calorimeters with Generative Adversarial Networks
- CCGAN Semi-Supervised Learning with Context-Conditional Generative Adversarial Networks
- CatGAN Unsupervised and Semi-supervised Learning with Categorical Generative Adversarial Networks
- CoGAN Coupled Generative Adversarial Networks

- Context-RNN-GAN Contextual RNN-GANs for Abstract Reasoning Diagram Generation
- C-RNN-GAN C-RNN-GAN: Continuous recurrent neural networks with adversarial training
- · CS-GAN Improving Neural Machine Translation with Conditional Sequence Generative Adversarial Nets
- CVAE-GAN CVAE-GAN: Fine-Grained Image Generation through Asymmetric Training
- · CycleGAN Unpaired Image-to-Image Translation using Cycle-Consistent Adversarial Networks
- DTN Unsupervised Cross-Domain Image Generation
- . DCGAN Unsupervised Representation Learning with Deep Convolutional Generative Adversarial Networks
- · DiscoGAN Learning to Discover Cross-Domain Relations with Generative Adversarial Networks
- DR-GAN Disentangled Representation Learning GAN for Pose-Invariant Face Recognition
- DualGAN DualGAN: Unsupervised Dual Learning for Image-to-Image Translation
- EBGAN Energy-based Generative Adversarial Network
- f-GAN f-GAN: Training Generative Neural Samplers using Variational Divergence Minimization
- FF-GAN Towards Large-Pose Face Frontalization in the Wild
- . GAWWN Learning What and Where to Draw
- GeneGAN GeneGAN: Learning Object Transfiguration and Attribute Subspace from Unpaired Data
- · Geometric GAN Geometric GAN
- · GoGAN Gang of GANs: Generative Adversarial Networks with Maximum Margin Ranking
- GP-GAN GP-GAN: Towards Realistic High-Resolution Image Blending
- IAN Neural Photo Editing with Introspective Adversarial Networks
- iGAN Generative Visual Manipulation on the Natural Image Manifold
- IcGAN Invertible Conditional GANs for image editing
- ID-CGAN Image De-raining Using a Conditional Generative Adversarial Network
- Improved GAN Improved Techniques for Training GANs
- · InfoGAN InfoGAN: Interpretable Representation Learning by Information Maximizing Generative Adversarial Nets
- LAGAN Learning Particle Physics by Example: Location-Aware Generative Adversarial Networks for Physics Synthesis
- · LAPGAN Deep Generative Image Models using a Laplacian Pyramid of Adversarial Networks

https://github.com/hindupuravinash/the-gan-zoo

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## "The GAN Zoo"

# See also: <a href="https://github.com/soumith/ganhacks">https://github.com/soumith/ganhacks</a> for tips and tricks for trainings GANs

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https://github.com/hindupuravinash/the-gan-zoo

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## **GANs**

Don't work with an explicit density function Take game-theoretic approach: learn to generate from training distribution through 2-player game

#### Pros:

- Beautiful, state-of-the-art samples!

#### Cons:

- Trickier / more unstable to train
- Can't solve inference queries such as p(x), p(z|x)

#### Active areas of research:

- Better loss functions, more stable training (Wasserstein GAN, LSGAN, many others)
- Conditional GANs, GANs for all kinds of applications

## Recap

### **Generative Models**

- PixeIRNN and PixeICNN Explicit density model, optimizes exact likelihood, good samples. But inefficient sequential generation.
- Variational Autoencoders (VAE) Optimize variational lower bound on likelihood. Useful latent representation, inference queries. But current sample quality not the best.
- Generative Adversarial Networks (GANs) Game-theoretic approach, best samples!
  But can be tricky and unstable to train,
  no inference queries.

## Recap

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  But can be tricky and unstable to train,
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Also recent work in combinations of these types of models! E.g. Adversarial Autoencoders (Makhanzi 2015) and PixelVAE (Gulrajani 2016)

## Recap

### **Generative Models**

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**Next time: Reinforcement Learning**