Advanced graph theory: Tutorial 4: CS60047 Autumn October 07, 2022

1. Show that the edge chromatic number for a graph G is $\chi'(G) \leq 2\Delta - 1$. Show also that for $\Delta(G) \geq 3$ this can be improved to $\chi'(G) \leq 2\Delta - 2$.

2. Show that $\chi'(G) \geq \Delta(G)$ for graph G. Show also that $\chi'(G) \geq \lceil \frac{E(G)}{\alpha'(G)} \rceil$, where $E(G)$ is the number of edges of G.

3. Show that $\chi'(G) = \Delta(G)$ for bipartite graph G. [Konig 1916].

4. Suppose graph G is 3-connected, regular, but not a complete graph. Show that there is a vertex u in G such that there exist two vertices v, w that are non-adjacent vertices in the neighbourhood of u.

5. Let x be any vertex of a 2-connected graph G, such that G is not a complete graph and $G - x$ is separable. Show that x is adjacent to two vertices y and z, that belong to two different *blocks* of $G - x$, where y and z are not adjacent.

6. Suppose $|V(G)| = n$ and $V(G)$ has a partition $\{V_1, V_2, ..., V_k\}$ such that for each $1 \leq i < j \leq k$, there exists an $x \in V_i$ and a $y \in V_j$ which are non-adjacent. Then $\chi(G) \leq n - k + 1$.

7. For an n-vertex graph G, show that $\chi(G) + \chi(\bar{G}) \leq n+1$, and $\chi(G).\chi(\bar{G}) \geq n$.