Advanced graph theory: Homework 3: CS60047 Autumn 2022

Instructor: Sudebkumar Prasant Pal
Teaching assistants: Aayushi Vidyanta and Karnam Sai Keerthana

September 11, 2022

Advanced graph theory: Homework 3: CS60047

Autumn 2022

1. Given the graph of 12 vertices with 8 edges 1,7;1,9;2,9;3,8;4,12;5,10;6,10;2,11; find α, β, α', χ, and the number of connected components.

2. (Reading and writing exercise) Take a bipartite graph $G(L \cup R, E)$ with $2n$ vertices in all, $|R| = |L| = n$. Its degree must be d for vertices in L and for every subset S of L of size at most $\alpha|L|$, we must have at least $c|S|$ neighbours in R. We will see that for large n, $d = 18$, $\alpha = \frac{1}{3}$ and $c = 2$, there is a solution. See the text Randomized Algorithms by Motwani and Raghavan, Cambridge University Press.

3. Show that a simple undirected graph of m edges and n vertices avoiding a $K_{r,r}$ as a subgraph satisfies $m < Cn^{2-\frac{1}{r}}$, where C depends only on r.

   [Hint: Use the technique as in Problem 4 of Tutorial 2.]

4. Draw a minimal connected simple graph that has no 1-factor but is 3-connected.

   [Hint: First show that any 3-connected graph with a 1-factor must contain every cut edge.]

5. Turan's theorem in the "Pigeonhole principle mode" may stated as follows. Let $G(V, E)$ be a graph with mk vertices and more than $\binom{k}{2}m^2$ edges. Then G must have a K_{k+1}.

   [Hint: See the technique used as shown in the slides for Turan's problem and use induction on m.]

6. Show that interval graphs as well as their complements are perfect.

   [Given a set of closed intervals on a line, assign a vertex for each interval and an edge for every pair of intersecting intervals to define the interval graph. Sweep a line perpendicular the intervals.]

7. Show that the intersection graph of subtrees of a tree is a chordal graph.

8. Show that every minimal (by inclusion) vertex separator subset in a chordal graph is a complete graph.

   [Show that every component of $G - S$ for a minimal separator S, is adjacent to every vertex in S. Also, show that any two vertices u and v in S appear in a 4-cycle.]