Advanced graph theory: Homework 1: CS60047

Autumn 2022

1. Show that at least two of all persons attending a party must have the same number of friends amongst the attendees.

2. Show that the number of pairs of friends is the half the sum of the numbers of friends of all persons.

3. Show in any planar drawing without crossings for a graph, we can start drawing a spanning cycle first, in a closed loop, and then draw internal and external chords for remaining edges judiciously, without crossings. Thereby show that K_5 and $K_{3,3}$ are not planar graphs.

4. Use induction to establish Euler's formula for the number of edges, vertices and faces of a planar drawing of a graph. Then show that for an $n(\geq 3)$-vertex graph, the number e of edges is at most $3n - 6$.

5. Use the necessary condition of at most $3n - 6$ edges in an n-vertex graph to show that K_5 is not planar.

6. Is the Petersen graph planar? Why?