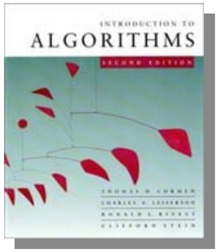
CS60020: Foundations of Algorithm Design and Machine Learning

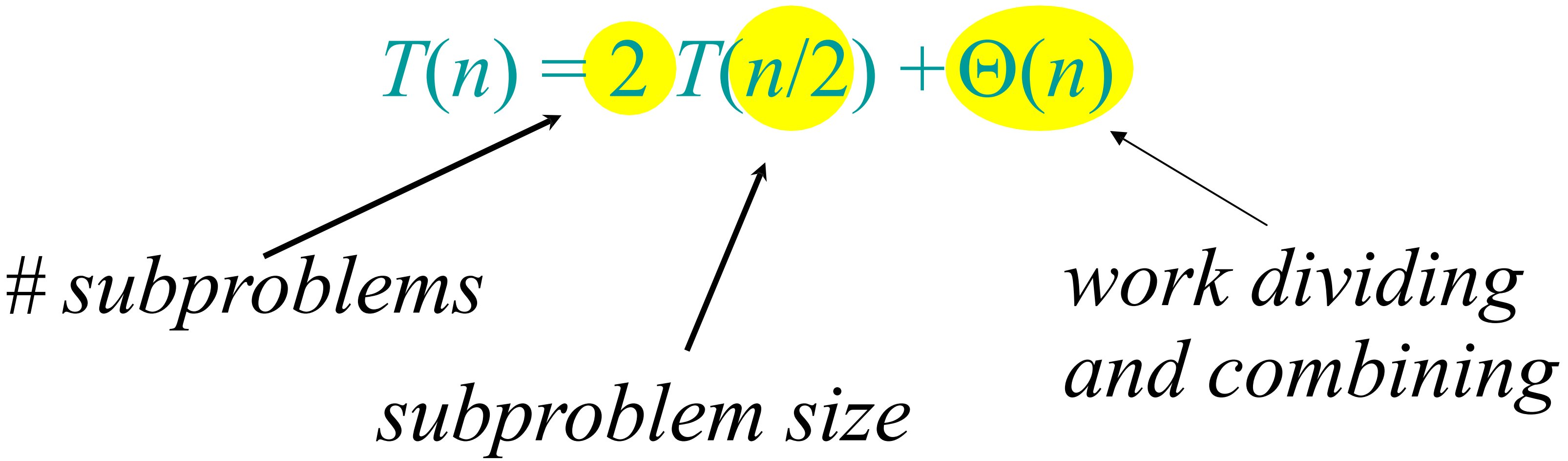
Sourangshu Bhattacharya

DIVIDE AND CONQUER



Merge sort

1. ***Divide:*** Trivial.
2. ***Conquer:*** Recursively sort 2 subarrays.
3. ***Combine:*** Linear-time merge.

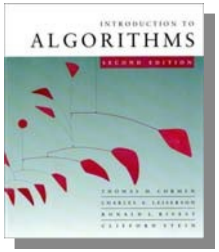


$$T(n) = 2\,T(n/2) + \Theta(n)$$

# subproblems

subproblem size

work dividing and combining



Master theorem

$$T(n) = a\,T(n/b) + f(n)$$

CASE 1: $f(n) = O(n^{\log_b a - \varepsilon})$, constant $\varepsilon > 0$

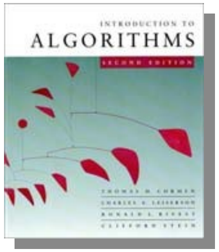
$\Rightarrow T(n) = \Theta(n^{\log_b a})$.

CASE 2: $f(n) = \Theta(n^{\log_b a})$

$\Rightarrow T(n) = \Theta(n^{\log_b a} \lg n)$.

CASE 3: $f(n) = \Omega(n^{\log_b a + \varepsilon})$, constant $\varepsilon > 0$, and regularity condition

$\Rightarrow T(n) = \Theta(f(n))$.



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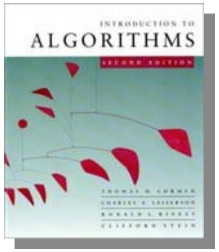
Merge sort: $a = 2$, $b = 2$ $\Rightarrow$ $n^{\log_b a} = n^{\log_2 2} = n$

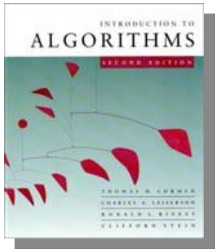
$\Rightarrow$ CASE 2 $\Rightarrow$ $T(n) = \Theta(n \lg n)$.



Binary search

- Find an element in a sorted array:

1. ***Divide:*** Check middle element.
2. ***Conquer:*** Recursively search 1 subarray.
3. ***Combine:*** Trivial.



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- ***Example:*** Find 9

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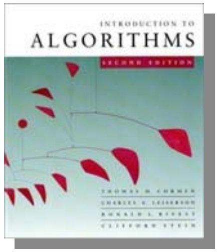

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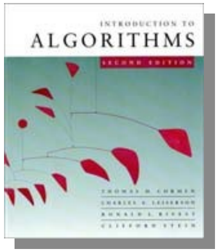

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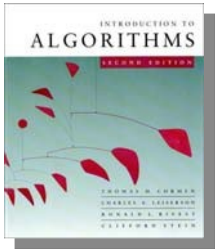

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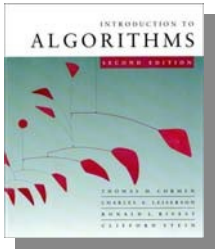

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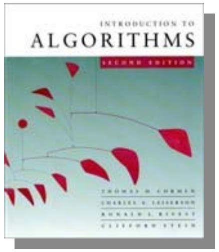

Recurrence for binary search

$$T(n) = 1\,T(n/2) + \Theta(1)$$

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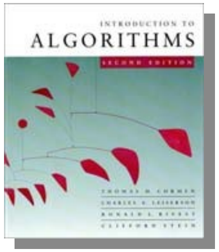

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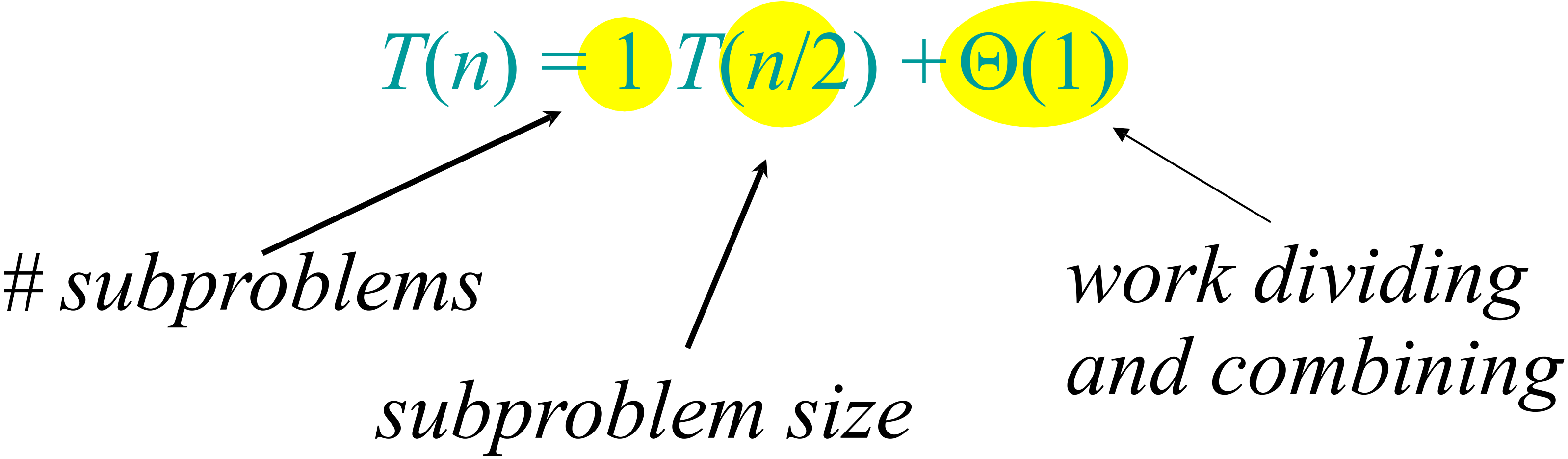

$$T(n) = 1\,T(n/2) + \Theta(1)$$

# subproblems

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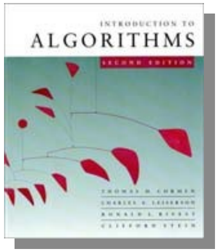
$$n^{\log_b a} = n^{\log_2 1} = n^0 = 1 \Rightarrow \quad \text{CASE 2 } (k = 0)$$

$$\Rightarrow \quad T(n) = \Theta(\lg n)\,.$$

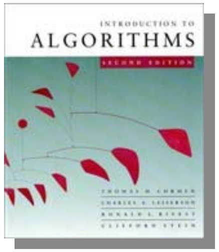

INTRODUCTION TO ALGORITHMS

SECOND EDITION

Powering a number

Problem: Compute a^n, where $n \in N$.

Naive algorithm: $\Theta(n)$.



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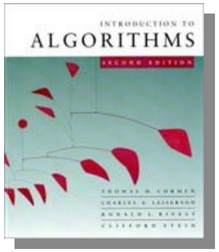
Divide-and-conquer algorithm:

$$a^n = \begin{cases} a^{n/2} \cdot a^{n/2} & \text{if } n \text{ is even;} \\ a^{(n-1)/2} \cdot a^{(n-1)/2} \cdot a & \text{if } n \text{ is odd.} \end{cases}$$



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$$T(n) = T(n/2) + \Theta(1) \;\Rightarrow\; T(n) = \Theta(\lg n)\,.$$