Lecture 5: Vehicular Control

Vehicular Control

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Module "Vehicle-2-X: Communication and Control"
Contents

- Basics of Control Theory
- Vehicular Control
Control Theory

- Open-loop control vs. Closed-loop control
  - Open-loop control: Control action of the controller is independent of the "process output" (no feedback)
  - Closed-loop control: Control action of the controller is dependent on feedback from the process in the form of the value of the process output

Source: Control theory, Wikipedia
Open-Loop vs Closed-Loop

- Vehicle cruise control example
  - Open-loop control
    - Lock the throttle position (recall the figure below?): controls the air intake
    - Vehicle will travel slower when climbing uphill
    - Cannot compensate for changes in circumstances
  - Closed-loop control
    - Data from sensor monitoring the vehicle speed
    - Controller continuously compared the sensor output with the desired speed
    - The „error“ determines the throttle position

Source: W. Ribbens, “Understanding automotive electronics”
Definitions

- **Sensor**: Component for measurement of a variable (signal)
- **Plant (or system)**: Is the part to be controlled
- **Controller**: Provides the satisfactory characteristics for the total system

- **Two types of control systems**
  - **Regulator**: Maintains a physical variable at some constant value in presence of perturbances
  - **Servomechanism**: A physical variable is required to follow or track some time-varying function
Control system is often described using block diagrams

Block diagrams contain *models*, a mathematical description of input-output relation of components combined with block diagram
A transfer function of a linear system is defined as the ratio of the Laplace transform of the output and the Laplace transform of the input:

\[ Y(s) = G(s)E(s) \]

\[ E(s) = R(s) - H(s)Y(s) \]

\[ Y(s) = G(s)[R(s) - H(s)Y(s)] \]

\[ \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \]

- System gain = forward gain / (1 + loop gain)

Source: http://bodetechnics.com/control-engineering-tutorials/transfer-function-block-diagram-manipulation/
Laplace Transform

- Laplace transform of $f(t)$ denoted by $F(s)$ or $L\{f(t)\}$, is an integral transform given by the Laplace integral:
  \[
  L\{f(t)\} = F(s) = \int_{0}^{\infty} e^{-st} f(t) dt
  \]
- Provided that this integral exists
- Transformation to the frequency domain is one-to-one
- $f(t) = 1$, $F(s) = \frac{1}{s}$
  \[
  L\{f(t)\} = \int_{0}^{\infty} e^{-st} dt = -\frac{1}{s} e^{-st} \bigg|_{0}^{\infty} = \frac{1}{s}
  \]
- $f(t) = t$, $F(s) = \frac{1}{s^2}$
- $f(t) = e^{at}$, $F(s) = \frac{1}{s-a}$
Some useful properties

- \( L\{f'(t)\} = sL\{f(t)\} - f(0) \)
- \( L\{f''(t)\} = s^2L\{f(t)\} - sf(0) - f'(0) \)
- \( L\{f'''(t)\} = s^3L\{f(t)\} - s^2f(0) - sf'(0) - f''(0) \)

Useful for solving linear differential equations

\[ y'' - 6y' - 5y = 0, \quad y(0) = 1, \quad y'(0) = -3 \]

Source: Z. S. Tseng, „The Laplace Transform“, 2008
Modeling of Dynamic System

- Dynamic system modeling example

- $k$: spring constant, $\gamma$: damping constant, $u(t)$: force

  \[
  \ddot{y} = -ky(t) - \gamma \dot{y}(t) + u(t) \\
  \dot{y}(t) + \gamma \dot{y}(t) + k y(t) = u(t) \\
  y(0) = y_0, \dot{y}(0) = \dot{y}_0
  \]

- This is an linear ordinary differential equation
  - Linear: no $y^2$
  - Ordinary: one independent variable
    (as opposed to partial differential equations)
Modeling of Dynamic System

- Express the highest order term
  \[ \ddot{y}(t) = -ky(t) - \gamma \dot{y}(t) + u(t) \]

- Put adder in front

- Synthesize all other terms using integrators
State Space Equation

- Any system which can be presented by LODE can be represented in **state space form** (matrix differential equation)
- Example
  \[ \ddot{y} = -ky(t) - \gamma \dot{y}(t) + u(t) \]
- Step 1: Deduce set off first order differential equation in variables 
  \( x_j(t) \): states of system 
  \( x_1(t) \): Position \( y(t) \) 
  \( x_2(t) \): Velocity \( \dot{y}(t) \) 
  \[ \begin{align*} 
  \dot{x}_1(t) &= \dot{y}(t) = x_2(t) \\
  \dot{x}_2(t) &= \ddot{y}(t) = -kx_1(t) - \gamma x_2(t) + u(t) 
  \end{align*} \]
- **One** linear ordinary differential equation (LODE) of order **two** is transformed into **two** LODE of order of **one**
State Space Equation

- Step 2: Put everything together in a matrix differential equation
\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-k & -\gamma
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} + \begin{bmatrix}
0 \\
1
\end{bmatrix} u(t)
\]

- State equation
\[\dot{x}(t) = Ax(t) + Bu(t)\]

- Measurement equation: related observed value to the state vector
\[y(t) = Cx(t) + Du(t)\]

- System state
  - **System state** $x$ of a system at any time $t_0$ is the “amount of information that together with all inputs for $t \geq t_0$, uniquely determines the behavior of the system for all $t \geq t_0$
State Space Equation

- Linear time-invariant (LTI) system is described by standard form of the state space equation
  \[ \dot{x}(t) = Ax(t) + Bu(t) \]
  \[ y(t) = Cx(t) + Du(t) \]

- In most cases D=0

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dimension</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x(t) )</td>
<td>( n \times 1 )</td>
<td>State vector</td>
</tr>
<tr>
<td>( A )</td>
<td>( n \times n )</td>
<td>System matrix</td>
</tr>
<tr>
<td>( B )</td>
<td>( n \times r )</td>
<td>Input matrix</td>
</tr>
<tr>
<td>( u(t) )</td>
<td>( r \times 1 )</td>
<td>Input vector</td>
</tr>
<tr>
<td>( y(t) )</td>
<td>( p \times 1 )</td>
<td>Output vector</td>
</tr>
<tr>
<td>( C )</td>
<td>( p \times n )</td>
<td>Output matrix</td>
</tr>
<tr>
<td>( D )</td>
<td>( p \times r )</td>
<td>Matrix representing direct coupling with input and output</td>
</tr>
</tbody>
</table>
Okay why bother with state space equations?
- Computers love state space equations
- Modern control uses state space equation
- Notations are not unique
  \[ y^{(n)}(t) + a_{n-1}y^{(n-1)}(t) + \cdots + a_1 y(t) + a_0 y(t) = b_m u^{(m)}(t) + \cdots + b_1 u(t) + b_0 u(t) \]
- Control-canonical form
  \[
  A = \begin{bmatrix}
  0 & 1 & 0 \\
  0 & 0 & 1 \\
  -a_0 & -a_1 & -a_2
  \end{bmatrix},
  B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},
  C = [b_0 \ b_1 \ b_2],
  D = b_3
  \]
- Observer-canonical form
  \[
  A = \begin{bmatrix}
  0 & 0 & -a_0 \\
  1 & 0 & -a_1 \\
  0 & 1 & -a_2
  \end{bmatrix},
  B = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix},
  C = [0 \ 0 \ 1],
  D = b_3
  \]
State Space Equation

Block diagrams:

Control-canonical Form:

Observer-Canonical Form:
State Space Equation

- Example

\[ \frac{d^2}{dt^2} y(t) + 4 \frac{d}{dt} y(t) + 3y(t) = 2u(t) \]

- State space equation
  - Let \( x_1(t) = y(t) \) and \( x_2(t) = \dot{y}(t) \)
  - \( \dot{x}_1(t) = \dot{y}(t) = x_2(t) \)
  - \( \dot{x}_2(t) + 4x_2(t) + 3x_1(t) = 2u(t) \)
  - \( x_1(t) = -3x_1(t) - 4x_2(t) + 2u(t) \)

- Write equations in matrix form
  - \( x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \)
  - \( \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t) \)
  - \( y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \)
Stability

- If we know transfer function $G(s)$, what can we say about the system stability?

- A linear time invariant system is called BIBO stable (bounded-input-bounded-output).

- For all bounded inputs $|u(t)| \leq M_1$ (for all $t$), exists a boundary for the output signal $M_2$, so that $|y(t)| \leq M_2$ for all $t$, with $M_1, M_2$, positive real numbers.
Example: \( Y(s) = G(s)U(s) \), interator \( G(s) = \frac{1}{s} \)

\[ \begin{align*}
    u(t) &= \delta(t), \quad U(s) = 1 \\
    |y(t)| &= |L^{-1}[Y(s)]| = \left| L^{-1} \left[ \frac{1}{s} \right] \right| = 1
\end{align*} \]

What happens when the input is \( u(t) = 1 \)?

\[ \begin{align*}
    u(t) &= 1, \quad U(s) = \frac{1}{s} \\
    |y(t)| &= |L^{-1}[Y(s)]| = \left| L^{-1} \left[ \frac{1}{s^2} \right] \right| = t
\end{align*} \]

(uncounted)

BIBO stability should be proven for ALL inputs
Y(s) = G(s)U(s)

By means of convolution theorem we get

\[ |y(t)| = |\int_0^t g(\tau)u(t-\tau)d\tau| \leq \int_0^t |g(\tau)||u(t-\tau)|d\tau \leq M_1 \int_0^t |g(\tau)d\tau \leq M_2 \]

Therefore,

If the impulse response, \( \int_0^\infty |g(\tau)d\tau < \infty \), is bounded, then the system is BIBO-stable

But what about transfer function?
Transfer Function of a State Space Model

- **State space model**

\[
\dot{x}(t) = Ax(t) + Bu(t)
\]
\[
y(t) = Cx(t) + Du(t)
\]

\[
sX(s) - x(0) = AX(s) + BU(s)
\]
\[
X(s) = (sI - A)^{-1}x(0) + (sI - A)^{-1}BU(s)
\]
\[
= \phi(s)x(0) + \phi(s)BU(s)
\]

\[
Y(s) = CX(s) + DU(s)
\]
\[
= C[(sI - A)^{-1}]x(0) + [C(sI - A)^{-1}B + D]U(s)
\]
\[
= C\phi(s)x(0) + C\phi(s)BU(s) + DU(s)
\]

- **Transfer function**

\[
G(s) = \frac{Y(s)}{U(s)} = C\phi(s)B + D
\]
Can stability be determined if we know the transfer function of a system?

State space model

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &=Cx(t) + Du(t)
\end{align*}
\]

The transfer function is given by

\[
H(s) = C (sI - A)^{-1} B + D = C \frac{\text{Adj}(sI - A)}{\det(sI - A)} + D
\]

- \(\text{Adj}(A)\) is the adjugate matrix of \(A\)
- When, \(A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\), then \(\text{Adj}(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\)

The poles of \(H(s)\) are uncancelled eigenvalues of \(A\), assuming \(D=0\)

- Let \(\lambda_i\) be the \(i^{th}\) eigenvalue of \(A\), if \(\lambda_i \leq 0, \forall i\), then system is stable
Vehicle Longitudinal Control: Sensor Inputs

- Four types of information are usually considered for the longitudinal control
  - Speed and acceleration of the host vehicle
  - Distance to the preceding vehicle
  - Speed and acceleration of the preceding vehicle
  - Acceleration and speed of the first vehicle (i.e., lead vehicle)

- Speed and acceleration of the host vehicle can be measured by speed sensors and accelerometers onboard the vehicle

- Distance to the preceding vehicle can be measured by ranging sensors, e.g., radar, LIDAR, ultrasonic sensors
  - Radar has been used most commonly
  - LIDAR is affected by weather (snow and fog)
Vehicle Longitudinal Control: Sensor Inputs

- Speed and acceleration of the preceding vehicle and lead vehicle
  - Speed and acceleration of the preceding vehicle can be derived from the host vehicle
    - However, requires differentiation of the radar sensor, which can be noisy
  - Communication
    - Transmit the speed and acceleration to the succeeding vehicle
    - Reliability of communication?
Assumptions

- Time delays associated with power generation in the engine are negligible
- Torque converter in the vehicle is locked
- No torsion in the drive axle
- Slip between the tires and the road is zero

Then, vehicle speed $V_x$ is directly related to the engine speed $\omega_e$

$$\dot{x} = v_x = Rh\omega_e$$

where $R$ and $h$ are gear ratio and tire radius
The simplified vehicle dynamics model takes three variables as states

- Mass of air in the intake manifold: $m_a$
- Engine speed: $\omega_e$
- Brake torque: $T_{br}$

The dynamics relating engine speed to the pseudo-inputs, net combustion torque $T_{net}$ and brake torque $T_{br}$ can then be modeled by

$$\dot{\omega} = \frac{T_{net} - c_a R^2 h^2 \omega_e^2 - R(h F_f + T_{br})}{J_e}$$

where $c_a$ is the aerodynamic drag coefficient, $F_f$ is the rolling resistance of the tires, and $J_e = I_e + (m h^2 + I_\omega) R^2$ is the effective inertia reflected on the engine side.
\( T_{net}(\omega_e, m_a) \) is a nonlinear function obtained from steady-state engine maps available from the vehicle manufacturer.
Dynamics relating $m_a$, the air mass flow in engine manifold, to the throttle angle can be modeled as

$$\dot{m}_a = \dot{m}_{ai} - \dot{m}_{ao}$$

where $m_{ai}$ and $m_{ao}$ are the flow rate into the intake manifold and out from the manifold

- $m_{ao}$ is a nonlinear function of $\omega_e$ and $P_m$, pressure of the air in engine manifold (from engine manufacturer)
- $m_{ai}$ is

$$\dot{m}_{ai} = MAX \cdot TC(\alpha)PRI(m_a)$$

where $MAX$ is a constant dependent on the size of the throttle body, $TC(\alpha)$ is a nonlinear invertible function of the throttle angle, and $PRI$ is the pressure influence function that describes the choked flow relationship which occurs through the throttle valve
How do we measure $m_a$? Ideal gas law

$P_m V_m = m_a R_g T$

Where $R_g$ is a variable that depends on the vehicle transmission gear ratio, and $T$ is the temperature.

Pressure can be measured to calculate $m_a$.
Brake model is linear and modeled by a first-order lag

\[ \tau_{br} \dot{T}_{br} + T_{br} = T_{br, cmd} = K_{br} P_{br} \]

where \( \tau_{br} \) is the brake system time constant, \( K_{br} \) is the total proportionality between the brake pressure \( P_{br} \) and the brake torque at the wheels.
Longitudinal Vehicle Model

- Simpler models
- Assumption
  - Vehicle reacts to the acceleration input without any delay (no inertia)
- Control input: acceleration
- Control output: inter-vehicle distance
  \[ \dot{v}(t) = u_i(t) \]
  \[ \dot{X}_i(t) = A_i X_i(t) + B_i U_i(t) \]
- What would be A and B in such a case?
  
  - \[ X_i(t) = \begin{bmatrix} x_i(t) \\ v_i(t) \end{bmatrix}, A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \]
  - \[ B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, U_i(t) = u_i(t) \]
  - But, in more recent works, more sophisticated models are used
Platooning vs Adaptive Cruise Control

- **Platooning**
  - Maintains constant distance
  - Requires communication among vehicles
  - Minimum traffic shockwave

- **Adaptive cruise control (ACC)**
  - Maintains minimum distance and time headway
  - Usually achievable with sensor inputs only
  - Already in commercial vehicles
Longitudinal Control System Architecture

- Usually consists of inner loop and outer loop controllers
- Outer loop controller (upper or high level) synthesizes a desired speed or acceleration
- Inner loop controller (lower level) will generate corresponding throttle or brake commands
Upper level controller

- Determines the desired acceleration for each vehicle so as to
  - Objective (1): Maintain constant small spacing between the vehicles
  - Objective (2) Ensure string stability of the platoon

- Plant model of the upper level controller is
  \[
  \ddot{x}_i = u_i
  \]
  where the subscript \( i \) denotes the \( i \)-th vehicle in the platoon

- However, due to the finite bandwidth associated with the lower level controller, each vehicle is actually expected to track the desired acceleration imperfectly
Performance specification of the upper level controller is therefore to meet objectives (1) and (2) in the presence of first-order lag in the lower level controller: time lag from the desired value ($\ddot{x}_{i,des}$)

\[
\ddot{x}_i = \frac{1}{\tau_s + 1} \dot{x}_{i,des} = \frac{1}{\tau_s + 1} u_i
\]

- This assumption is made to simplify the design of the high-level control
- (What’s first-order lag?)

\[
\tau \frac{dy}{dx} + y = x
\]

The spacing error for the i-th vehicle is defined as $\epsilon_i = x_i - x_{i-1} + L$, where $\epsilon_i$ is the longitudinal spacing error of the i-th vehicle, $L$ being the desired spacing
Spacing Policies

- Speed independent spacing policy (constant spacing)
  - \( x_{rd} = d_0 \)
  - Usually achievable only with v2v communication

- Speed-dependent spacing policy
  - Semi-autonomous: can be implemented with sensor measurements only
  - \( x_{rd} = d_0 + \dot{x}t_{hw} \)
  - Time headway is used
  - Commonly used in commercial ACC systems
  - \( d_0 \) is the minimum safe distance
  - \( t_{hw} \) is the time gap
  - Similar to driver’s daily experience
Different speed-dependent spacing policies have been proposed in literature:

- Time headway policy performs poor against traffic flow fluctuation (Remember the video from the introduction?)
- Nonlinear spacing policy for the stability of the traffic flow has been proposed

\[ x_{rd} = \frac{1}{\rho_m \left(1 - \frac{\dot{x}_i}{v_f}\right)} \]

where \(\rho_m\) is the traffic density parameter
The objectives (1) and (2) can be mathematically stated as
\[ \varepsilon_{i-1} \to 0 \Rightarrow \varepsilon_i \to 0 \]
\[ \left\| |H(s)| \right\|_{inf} \leq 1 \]

where \( \hat{H}(s) \) is the transfer function relating the spacing errors of consecutive vehicles in the platoon:
\[ \hat{H}(s) = \frac{\varepsilon_i(s)}{\varepsilon_{i-1}(s)} \]

- Notation \( \left\| |H(s)| \right\|_{inf} \) denotes the largest value \( H(s) \) could have according to changing value of \( i \) (vehicles)
The string stability of the platoon

- Practically, it means the gap regulation error will not be amplified from the lead vehicle to the last vehicle in the platoon
- $\|H(s)\|_{\infty} \leq 1$, refers to a property in which spacing errors are guaranteed to diminish as they propagate toward the tail of the platoon
- Example: Any errors between the second and third cars do not amplify into an extremely large spacing error between seventh and eight cars
- Will be robust against internal vehicle dynamics as well as the imperfections in the lower-loop

Traffic network point of view on string stability

- Less shockwaves

Driver’s point of view

- Smooth ride and safety benefit
Upper Level Controller Design

- Linear Controller Design
  - Desired acceleration

\[ a_{d,i} = - \frac{1}{h(x_{r,i} + \lambda e_i)} \]
Example: Sliding mode control

- Nonlinear control method that alters the dynamics of a nonlinear system by application of a discontinuous control signal that forces the system to „slide“ along a cross-section of the system’s normal behavior
- Step 1: select a sliding surface (which is stable)
- Step 2: design a control law which will attract the system status to the sliding surface and remain there

Source: https://www.globalspec.com/reference/21394/160210/chapter-5-4-2-sliding-mode-control
Simple sliding mode control for ACC

Define the sliding surface

\[ S_1 = e_i = R_i - T_h v_i, \]
where \( e_i \) is the range error, \( R_i \) is the range of \( i \)-th vehicle, \( T_h \) is the time headway

Design a control law

\[ \dot{S}_1 = \dot{e}_i = \lambda S_1, \]
where \( \lambda \) is the convergence rate to the sliding surface

Actual input is acceleration, so if we re-arrange by substituting \( S \)

\[ a_i = \frac{\lambda}{T_h} e_i + \frac{1}{T_h} \dot{R}_i \]

What does it mean?

- Smaller time gap means aggressive control (larger acceleration)
- And higher risk of losing string stability \( T_h \geq 2\tau \)