# Tutorial 6
Turing Machines, Recursive and R.E. Sets

1. Design a total Turing machine that accepts the set $\{0^{2^i} \mid i \geq 1\}$.

2. A <u>linear bounded automaton</u> (LBA) is exactly like a 1-tape TM, except that the input string $x \in \Sigma^*$ is enclosed in left and right endmarkers $\vdash$ and $\dashv$ which may not be overwritten. The machine is constrained never to move left of $\vdash$ or right of $\dashv$. It is allowed to read/write between these markers.

2-way read/write tape

Finite Control
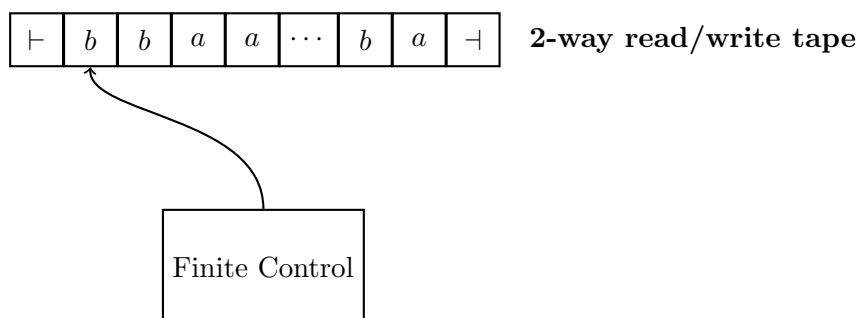
   (a) Give a rigorous formal definition of deterministic linearly bounded automata, including a definition of configurations and acceptance.

   (b) Let $\mathcal{M}$ be an LBA with state set Q of size k and tape alphabet Γ of size m. How many possible configurations are there on input x with $|x| = n$?

   (c) Argue that it is possible to detect in finite time whether an LBA loops on a given input.
   Hint: Use part (b).

3. Show that the class of recursively enumerable sets is closed under union and intersection.

4. Prove that a language L is recursive if and only if there is an enumeration machine enumerating the strings of L in a non-decreasing order of length (string of the same length may be arranged in lexicographic order). For example, strings of $0, 1^*$ would be arranged as $0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 100, 101, 110, 111, \ldots$.

5. Is the set $\{\mathcal{M} \mid \mathcal{L}(\mathcal{M}) \text{ contains atleast 100 elements}\}$ recursively enumerable?