Tutorial 5

Context-Free Languages/Grammars and Pushdown Automata

1. Describe context-free grammars for the following languages over the alphabet $\{a, b, c\}$.
   (a) $\{a^i b^j c^k \mid i = j \text{ or } j = k\}$
   (b) $\{a^i b^j c^k \mid i + j = k\}$
   (c) $\{a^i b^j c^k \mid i + k = j\}$
   (d) $\{a^i b^j c^k \mid i + k < j\}$

   Design pushdown automata for the above languages. State whether your PDA accepts by empty stack or final state.

2. What context-free languages do the following context-free grammars G generate?
   $G = (\{S, A, B\}, \{a, b, c\}, P, S)$ with the set of production rules P is given by
   (a) $S \to ASB \mid \epsilon, \quad A \to a, \quad B \to bb \mid b$
   (b) $S \to abScB \mid \epsilon, \quad B \to bB \mid b$

3. Consider the following context-free grammar $G = (\{S, A, B\}, \{a, b\}, P, S)$ with P containing the following production rules.

$$S \to ABS \mid AB$$
$$A \to aA \mid a$$
$$B \to bA$$

   Which of the following strings are in $L(G)$ and which are not? Provide derivations for those that are in $L(G)$ and reasons for those that are not.
   (a) $aabaab$
   (b) $aaaaba$
   (c) $aabbaa$
   (d) $abaaba$

4. Provide grammars in Chomsky normal form for the following languages.
   (a) $\{a^n b^{2n} c^k \mid k, n \geq 1\}$.
   (b) $\{a, b\}^* - \{x \in \{a, b\}^* \mid x = x^{\mathbf{R}}\}$.
   (c) $\{a^n b^k c^n \mid k, n \geq 1\}$.

5. Which of the following languages are context-free? Justify - if context-free define a CFG or PDA for the language and otherwise prove using pumping lemma for CFLs.
   (a) $\{x \in \{0, 1\}^* \mid \bar{x} = x^{\mathbf{R}}\}$. Here $\bar{x}$ denotes the Boolean complement of the string x.
   (b) $\{a^k \mid k \text{ is a power of } 2\}$.
   (c) $\{a^n b^m c^k \mid n, m, k \geq 1 \text{ and } n + k = m\}$
   (d) $\{a^p \mid p \text{ is a prime}\}$.
   (e) $\{a^n b^m \mid n, m \geq 1 \text{ and } m \neq 2n\}$

6. A context-free grammar G is called *ambiguous* if some string in $L(G)$ has two different derivation/parse trees. Show that the grammar $G = (\{S\}, \{a, b, c\}, P, S)$ with P defined as

$$S \to aS \mid aSbS \mid c$$

is ambiguous. Construct an unambiguous grammaar G' with $L(G') = L(G)$.