Tutorial 3
Finite Automata and Regular Languages

1. Design a DFA for the following language: $\{x \in \{a,b\}^* \mid \#a(x) = 0 \bmod 2 \text{ and } \#b(x) = 0 \bmod 3\}$.
2. Consider the set $A = \{x \in \{a,b\}^* \mid x \text{ ends with 3 consecutive } b\text{'s}\}$.
   (a) Design an NFA for A.
   (b) Using subset construction, construct an equivalent DFA for the NFA from the previous part.
   (c) Reduce the number of states in the resulting DFA by removing unreachable states.
3. For a string x, $x^{\mathbf{R}}$ denotes the reverse of x. Define $A^{\mathbf{R}} = \{x^{\mathbf{R}} \mid x \in A\}$ for a set A. Prove or disprove: if A is regular then so is $A^{\mathbf{R}}$.
4. Give regular expressions for each of the following subsets of $\{a,b\}^*$.
   (a) $\{x \mid x \text{ contains an odd number of } a\text{'s}\}$.
   (b) $\{x \mid x \text{ contains an even number of } b\text{'s}\}$.
   (c) $\{x \mid x \text{ contains an odd number of } a\text{'s and an even number of } b\text{'s}\}$.
5. Which of the following are regular sets? Justify.
   (a) $\{a^n \mid n \text{ is a prime number}\}$
   (b) $\{x \in \{a,b\}^* \mid x \text{ does not have three consecutive occurences of } a\}$
   (c) $\{x \in \{a,b\}^* \mid x = x^{\mathbf{R}}\}$
   (d) $\{x \in \{a,b,c\}^* \mid \#b(x) = 4n+1, n \geq 1\}$