Problems 1-6 already discussed in class.

7. R defined on $\overbrace{\mathbb{Z} \times (\mathbb{Z} \setminus \{0\})}^{A}$ as

$((a,b),(c,d)) \in R$ iff $ad = bc$.

→ $ab = ba$ → $((a,b),(a,b)) \in R$

for all $(a,b) \in \mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$

R is reflexive.

→ for all $a, c \in \mathbb{Z}$, $b, d \in \mathbb{Z} \setminus \{0\}$

$((a,b),(c,d)) \in R$ iff $ad = bc$

iff $da = cb$ i.e, $cb = da$

iff $((c,d),(a,b)) \in R$

R is symmetric

→ Let $a, c, e \in \mathbb{Z}$ & $b, d, f \in \mathbb{Z} \setminus \{0\}$

$((a,b),(c,d)), ((c,d),(e,f)) \in R$

We have $ad = bc$ and $cf = de$.

$(a\cancel{d})(\cancel{c}f) = (b\cancel{c})(\cancel{d}e)$

$af = be$ i.e., $((a,b),(e,f)) \in R$

R is transitive.

Now consider any equivalence class $[(a,b)] \in A/R$. $[(a,b)]$ consist of all pairs (c,d) s.t. $ac=bd$ i.e., $\frac{a}{b} = \frac{c}{d}$ & so they all represent the same rational number $\frac{a}{b}$.

Also given any $t \in \mathbb{Q}$, $\exists a,b \in \mathbb{Z}$, $b \neq 0$ s.t. $t = \frac{a}{b}$ which corresponds to the equivalence class $[(a,b)] \in A/R$.

$\therefore$ A/R & $\mathbb{Q}$ are essentially the same.

8. (a) Straight forward to show $(\Sigma^*, \preceq)$ is a partial order.

It is a total order since $\forall x,y \in \Sigma^*$ either $x \preceq y$ or $y \preceq x$.

(b) ϵ (empty string) is the least element.

(c) There is no greatest element.

(d) Clearly $\Sigma^{l_1}, \Sigma^{l_2} \subseteq A$.

$\exists x_0 \in \Sigma^{l_1}$ s.t. $x_0 \preceq y$ $\forall y \in \Sigma^{l_1} \setminus \{x_0\}$.

This x_0 is the least element & the only minimal element.

111^{l_y}, $\exists\, x_1 \in \Sigma^{l_2}$ s.t. $y \leq x_1 \ \forall\, y \in \Sigma^{l_2} \setminus \{x_1\}$.
x_1 is both greatest & maximal.

(e) Let $a \in \Sigma$.
Lower bounds : $a^{l_1 - 2}$, $a^{l_1 - 1}$
Upper bounds : $a^{l_2 + 1}$, $a^{l_2 + 2}$

x_0, x_1 from previous part are resp.
greatest lower bound & least upper bound.

9. Define $f : 2^S \to (S \to \{0,1\})$

set of all Boolean functions on S.

For $A \in 2^S$, define $f(A) = \chi_A$.

<u>f is onto:</u> Given a characteristic fn.
χ_A, define $A = \{x \in S \mid \chi_A(x) = 1\}$
A is the pre-image of χ_A & $A \in 2^S$
$\Rightarrow f$ is onto.

<u>f is 1-1:</u> Suppose $f(A) = f(B)$
$\chi_A(x) = \chi_B(x) \quad \forall x \in S.$

That is

$$\{x \in S \mid \chi_A(x) = 1\} = \{x \in S \mid \chi_B(x) = 1\}$$

i.e., $A = B$. $\Rightarrow f$ is 1-1.

$\therefore f$ is a bijection.

10. (a) NO.

$f(4) = f(-4) = 4 \Rightarrow$ not 1-1.

(b) NO.

$f(0) = f(19) = 0 \Rightarrow$ not 1-1

(c) NO. (why?)

[Verify whether $f : \mathbb{R} \to \mathbb{R}$ $f(x) = x^3$ is a bijection].

(d) YES.

$f^{-1} = f$