Tutorial 1
Propositional and Predicate Logic

Propositional Logic

1. Which of the following statements are tautologies, satisfiable or unsatisfiable?
   (a) $p \to p$
   (b) $p \to q$
   (c) $p \land q \to \neg q$
   (d) $p \lor \neg p \lor q$
   (e) $p \land \neg p \land q$
   (f) $(p \to q) \to (\neg q \to \neg p)$
   (g) $(p \to q) \to ((p \land r) \to q)$
   (h) $((p \land r) \to q) \to (p \to q)$

2. Let p, q, r, s be atomic propositions. Is there any truth assignment to p, q, r, s that make all the following statements true?

$$p \leftrightarrow q, \quad q \to r, \quad \neg r \lor s, \quad \neg p \to s, \quad \neg s$$

3. Encode and reason about the following.

   If a scarcity of commodities develops, then the prices rise. If there is a change of government, then fiscal controls will not be continued. If the threat of inflation persists, then fiscal controls will be continued. If there is over-production, then prices do not rise. It has been found that there is over-production and there is a change of government. Therefore, neither the scarcity of commidities has developed, nor there is a threat of inflation.

4. Prove the following logical deductions.

   (a)

$$\begin{array}{l} t \to q \\ \neg r \to \neg s \\ p \to u \\ \neg t \to \neg r \\ u \to s \\ \hline \therefore p \to q \end{array}$$

   (b)

$$\begin{array}{l} p \land q \to r \\ s \land \neg t \\ s \land \neg(t \lor p) \to u \\ \hline \therefore q \land \neg r \to u \end{array}$$

(c)

$$(p \to q) \to r$$
$$s \to \neg p$$
$$t$$
$$\neg s \wedge t \to q$$
$$\therefore r$$

(d)

$$\neg p \vee q \to r$$
$$r \to s \vee t$$
$$\neg s \wedge \neg u$$
$$t \to u$$
$$(q \to v) \wedge (v \to q)$$
$$(v \wedge w) \vee (\neg v \wedge w) \to \neg p$$
$$\therefore \neg w$$

5. Encode and deduce using logical inferencing techniques.

   While walking in a labyrinth, you find yourself in front of three possible roads. The road on your left is paved with gold, the road in front of you is paved with marble, while the road on your right is made of small stones. Each road is protected by a guard. You talk to the guards, and they tell you this:

   - *The guard of the gold road: "This road will bring you straight to the center. Moreover, if the stones take you to the center, then also the marble takes you to the center."*
   - *The guard of the marble road: "Neither the gold nor the stones will take you to the center."*
   - *The guard of the stone road: "Follow the gold, and you will reach the center. Follow the marble, and you will be lost."*

   You know that all the guards are liars. Your goal is to choose the correct road that will lead you to the center of the labyrinth.

Predicate Logic

1. Assume that the following predicates/formulas are defined (over the set of natural numbers $\mathbb{N} = \{0, 1, 2, \cdots\}$):

   - $\mathsf{Div}(x, y)$: true if x divides y and false otherwise
   - $x \geq y$: true if x greater than or equal to y and false otherwise
   - $x \neq y$: true iff x is not equal to y

   Write down formulas in predicate logic for the following.

   (a) $\mathsf{Prime}(p)$ which should be true iff p is a prime number

   (b) "There are infinitely many primes".

   (c) "There are infinitely many twin primes." Twin primes are pairs of primes that differ by 2.

   (d) "For $n \geq 3$, the equation $x^n + y^n = z^n$ has no postive integer solutions." This is the famous Fermat's last theorem.

2. Formalise the following sentences in predicate logic using only the following predicates.

   - $\mathsf{Admires}(x, y)$: x admires y
   - $\mathsf{Attend}(x, y)$: x attended y

- $\mathsf{Prof}(x)$: x is a professor
- $\mathsf{Stu}(x)$: x is a student
- $\mathsf{Lec}(x)$: x is a lecture

(a) Mohan admires every professor.

(b) Some professor admires Mohan.

(c) Mohan admires himself.

(d) No student attended every lecture.

(e) No lecture was attended by every student.

(f) No lecture was attended by any student.

3. Encode the following logical statements using predicate logic (formulate suitable predicate and function symbols as required), and conclude on the validity of the last statement.

   (a) Every athlete is strong. Everyone who is strong and intelligent will succeed in career. Hima is an athlete. Hima is intelligent. Therefore, Hima will succeed in career.

   (b) No man who is a candidate will be defeated if he is a good campaigner. Any man who runs for office is a candidate. Any candidate who is not defeated will be elected. Every man who is elected is a good campaigner. Therefore, Any man who runs for office will be elected if and only if he is a good campaigner.

   (c) Jack owns a dog. Every dog owner is an animal lover. No animal lover kills an animal. Either Jack or Curiosity killed Tuna, which is a cat. Did Curiosity kill the cat?

   (d) Everything that dies decomposes. Everything that decomposes divides into parts. Only material things are divisible into parts. No human souls are material. Therefore, no human souls die.

4. Translate the following into idiomatic/concise English statements.

   (a) $\forall x\,((\mathsf{H}(x) \wedge \forall y \neg \mathsf{M}(x,y)) \rightarrow \mathsf{U}(x))$ where $\mathsf{H}(x)$ means x is a man, $\mathsf{M}(x,y)$ means x is married to y, $\mathsf{U}(x)$ means x is unhappy and x, y range over people.

   (b) $\exists z \exists x\,(\mathsf{P}(z,x) \wedge \forall y(\mathsf{S}(z,y) \wedge \mathsf{W}(y)))$ where $\mathsf{P}(z,x)$ means z is parent of x, $\mathsf{S}(z,y)$ means z and y are siblings, $\mathsf{W}(y)$ means y is a woman and x, y, z range over people.

5. For each of the following equivalences, show a proof using deduction rules.

   (a) $\forall xp \wedge q \simeq \forall x(p \wedge q)$

   (b) $\exists x(p \rightarrow q) \simeq \forall xp \rightarrow q$

   (c) $\forall x(p \rightarrow q) \simeq \psi \rightarrow \forall x\phi$