PROPOSITIONAL LOGIC

1. (a) Tautology
   (b) Satisfiable
   (c) Satisfiable
   (d) Tautology
   (e) Unsatisfiable
   (f) Tautology
   (g) Tautology
   (h) Satisfiable

2. No!
   We want all statements true.
   So we have $s = F$.
   To make $\neg r \vee s$, $\neg p \to s$ true, set
   $p = T$, $r = F$.
   Since $q \to r$, set $q = F$.
   But now, $p \leftrightarrow q$, $q = F$ & $p = T$
   contradiction!
   $\therefore$ there is no satisfying assignment.

3. p: A scarcity of commodities develops.

q: Prices rise

r: There is change of government.

s: Fiscal controls will be continued.

t: There is over-production.

u: There is a threat of inflation.

Given

$p \to q$

$r \to \neg s$

$u \to s$

$t \to \neg q$

$t \wedge r$

$\therefore \neg p \wedge \neg u$

①

$$\dfrac{t \wedge r}{t} \ \wedge \text{elimination}$$

$$\dfrac{t \qquad t \to \neg q}{\neg q} \ \text{Modus ponens}$$

$$\dfrac{\neg q \qquad p \to q}{\neg p} \ \text{Modus tollens}$$

②

$$\dfrac{t \wedge r}{r} \ \wedge \text{elimination}$$

$$\dfrac{r \qquad r \to \neg s}{\neg s} \ \text{modus ponens}$$

$$\dfrac{\neg s \qquad u \to s}{\neg u} \ \text{modus tollens}$$

From ① & ②, we have

$\neg p \wedge \neg u$,

and hence the given argument is correct.

4. (c) $t \qquad \neg s \wedge t \to q$

$\neg s$ assumption

$\neg s \wedge t$ $\wedge$ introduction

q $\to$ elimination

$\neg s \to q$ $\to$ introduction

$\neg s \xrightarrow{2} q \qquad \dfrac{s \to \neg p}{p \xrightarrow{1} \neg s}$ contrapositive

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p assumption

$\neg s$ $\xrightarrow{1}$ elimination

q $\xrightarrow{2}$ elimination

$p \to q$ $\to$ introduction

$$\frac{p \to q \qquad (p \to q) \to r}{r}$$ modus ponens

(d)

$$\frac{\neg s \wedge \neg u}{\neg u} \ \wedge \text{ elimination}$$

$$\frac{\neg u \qquad t \to u}{\neg t} \ \text{modus tollens}$$

$$\frac{\neg s \wedge \neg u}{\neg s} \ \wedge \text{ elimination}$$

$$\frac{\neg t \qquad \neg s}{\neg (s \vee t)} \ \vee \text{ introduction}$$

$$\frac{\neg (s \vee t) \qquad r \to s \vee t}{\neg r} \ \text{modus tollens}$$

$$\frac{\neg r \qquad \neg p \vee q \to r}{p \wedge \neg q} \ \text{modus tollens}$$

$$\frac{(q \to v) \wedge (v \to q)}{v \to q} \ \wedge \text{ elimination}$$

$$\frac{p \wedge \neg q}{\neg q}$$

$$\frac{v \to q \qquad \neg q}{\neg v} \ \text{modus tollens}$$

$$\frac{p \wedge \neg q}{p} \ \wedge \text{ elimination}$$

$$\frac{p \qquad (v \wedge w) \vee (\neg v \wedge w) \to \neg p}{\neg((v \wedge w) \vee (\neg v \wedge w))} \ \text{modus tollens}$$

$$\frac{\neg((v \wedge w) \vee (\neg v \wedge w))}{(\neg v \vee \neg w) \wedge (v \vee \neg w)} \ \text{DeMorgan's rules}$$

$$\frac{(\neg v \vee \neg w) \wedge (v \vee \neg w)}{v \vee \neg w} \ \wedge \text{ elimination}$$

$$\frac{v \vee \neg w \qquad \neg v}{\neg w} \ \text{modus tollens}$$

PREDICATE LOGIC

1. (a) Prime(p) :

$$p \neq 1 \vee (\forall x\, (Div(x,p) \rightarrow (x=1 \vee x=p)))$$

(b) $\forall n \;\; \exists p \; (p \geq n \wedge Prime(p))$

(c) $\forall n \; \exists p \; (p \geq n \wedge Prime(p) \wedge Prime(p+2))$

(d) $\forall n \; ((n \geq 3) \rightarrow \neg(\exists x \exists y \exists z (x \geq 1 \wedge y \geq 1 \wedge z \geq 1 \wedge x^n + y^n = z^n)))$

2. (a) Let m be a constant representing Mohan.

$$\forall x \; (Prof(x) \rightarrow Admires(m, x))$$

(d) $\forall x \; (Stu(x) \rightarrow \exists y \, (Lec(y) \wedge \neg Attend(x,y)))$

3. (b) $C(x)$: x is a candidate

$O(x)$: x runs for office

$G(x)$: x is a good campaigner.

$E(x)$: x is elected.

$D(x)$: x is defeated

Given premises

$$\forall x \, (C(x) \to (G(x) \to \neg D(x)))$$

$$\forall x \, (\neg D(x) \to E(x))$$

$$\forall x \, (O(x) \to C(x))$$

$$\forall x \, (E(x) \to G(x))$$

Given conclusion

$$\forall x \, (O(x) \to (E(x) \leftrightarrow G(x)))$$

We have

$$\frac{\begin{array}{l}\forall x \, (O(x) \to C(x)) \\ \forall x \, (C(x) \to (G(x) \to \neg D(x)))\end{array}}{\forall x \, (O(x) \to (G(x) \to \neg D(x)))}$$

x_0 → arbitrary (→ elimination)

$O(x_0)$

$$\frac{G(x_0) \to \neg D(x_0) \qquad \neg D(x_0) \to E(x_0)}{G(x_0) \to E(x_0)}$$

$$\forall x\,(E(x) \to G(x))$$

$$\frac{G(x_0) \to E(x_0) \qquad E(x_0) \to G(x_0)}{E(x_0) \leftrightarrow G(x_0)}$$

→ introduction

$$\forall x\,(O(x) \to (E(x) \leftrightarrow G(x)))$$

4. (a) Every unmarried man is unhappy.

5. (b) $\exists x(p \to q)$

x_0 → one value for which $p \to q$

∃ elimination $\quad p[x_0/x] \to q[x_0/x]$

$$\frac{\forall x\, p}{p[x_0/x]} \quad \forall \text{elimination}$$

$$\frac{p[x_0/x] \to q[x_0/x] \qquad p[x_0/x]}{q[x_0/x]} \quad \text{modus ponens}$$

$$\forall x\, p \to q$$