

CS60088 Foundations of Cryptography

Spring 2026

Short Test 1

16-February-2026, 3 PM – 3:40 PM

Marks = 10

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*Here are all questions given in short test 1.*

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**QUESTION 1:** Let  $G$  be a PRG that stretches  $n$ -bit strings to  $3n$ -bit strings. For  $s \in \{0, 1\}^n$ , write  $G(s) = G_0(s) \| G_1(s) \| G_2(s)$ , so that  $G_0(s)$ ,  $G_1(s)$  and  $G_2(s)$  represent the first, second and third  $n$ -bit components of  $G(s)$  respectively. Define a new PRG  $G'$  that stretches  $n$ -bit strings to  $9n$ -bit strings as:

$$G'(s) = G(G_0(s)) \| G(G_1(s)) \| G(G_2(s)).$$

Prove or Disprove: if  $G$  is a secure PRG, then so is  $G'$ .

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**QUESTION 2:** Let  $\Pi_1 = (\text{Gen}_1, \text{Mac}_1, \text{Vrfy}_1)$  and  $\Pi_2 = (\text{Gen}_2, \text{Mac}_2, \text{Vrfy}_2)$  be 2 MAC schemes. Construct a MAC scheme  $\Pi = (\text{Gen}, \text{Mac}, \text{Vrfy})$  as follows:

$\text{Gen}(1^\lambda)$ : return  $(k_1, k_2)$  where  $k_1 \leftarrow \text{Gen}_1(1^\lambda)$  and  $k_2 \leftarrow \text{Gen}_2(1^\lambda)$

$\text{Mac}((k_1, k_2), m)$ : return  $t = (\text{Mac}_1(k_1, m), \text{Mac}_2(k_2, m))$

$\text{Vrfy}((k_1, k_2), m, t = (t_1, t_2))$ : return 1 if and only if  $\text{Vrfy}_1(k_1, m, t_1) = 1$  and  $\text{Vrfy}_2(k_2, m, t_2) = 1$

Prove or Disprove:  $\Pi$  is secure if either  $\Pi_1$  is secure or  $\Pi_2$  is secure.

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**QUESTION 3:** Suppose that  $h : \{0, 1\}^{2n} \rightarrow \{0, 1\}^n$  is a collision resistant hash function. Define a hash function  $H : \{0, 1\}^{4n} \rightarrow \{0, 1\}^n$  as follows: write  $x \in \{0, 1\}^{4n}$  as  $x = x_1 \| x_2$  with  $x_1, x_2 \in \{0, 1\}^{2n}$  and define

$$H(x) = h(h(x_1) \| h(x_2)).$$

Prove or Disprove:  $H$  is collision-resistant.

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