Indian Institute of Technology Kharagpur
Department of Computer Science and Engineering

CS60025 Algorithmic Game Theory **Autumn 2026**

Class Test 2 **13 November, 2:30 PM – 3:30 PM** **Marks = 20**

Answer all questions. State all assumptions you make. Keep your answers concise.

1. Recall that price of anarchy for a Pigou-like network with cost functions from a class $\mathcal{C}$ is given by
$$\alpha(\mathcal{C}) = \sup_{c\in\mathcal{C}} \sup_{r\geq 0} \sup_{y\in[0,r]} \frac{r\cdot c(r)}{(r-y)\cdot c(r) + y\cdot c(y)}.$$
   (a) If $\mathcal{C}$ is the class of affine functions, consisting of cost functions of the form $c(x) = ax + b$ for some real numbers $a, b \geq 0$, then show that $\alpha(\mathcal{C}) = \frac{4}{3}$.
   (b) Let $\mathcal{C}$ be the class of non-negative non-decreasing concave functions. Show that $\alpha(\mathcal{C}) = \frac{4}{3}$. *A function $f : \mathbb{R} \to \mathbb{R}$ is concave if for all $x, x' \in \mathbb{R}$ and any $\lambda \in [0, 1]$, $f(\lambda x + (1-\lambda)x') \geq \lambda f(x) + (1-\lambda)f(x')$.*
   Hint: Show that every non-negative non-decreasing concave function is bounded below by an affine function.

6+4 = 10

2. Let G be any instance of the load balancing game with three tasks that should be placed on two identical machines. Prove or Disprove: $\mathsf{cost}(A) = \mathsf{opt}(G)$ for any pure Nash equilibrium assignment A.

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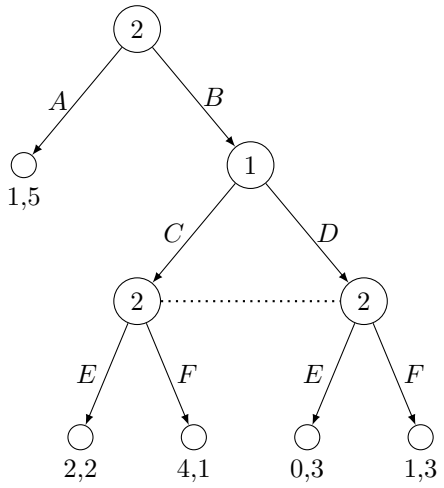
3. For the game shown below write down the terminal histories, proper subhistories, information sets and convert to a strategic/normal form game.



1+1+1+2 = 5