Indian Institute of Technology Kharagpur
Department of Computer Science and Engineering

CS60025 Algorithmic Game Theory **Autumn 2026**

Class Test 1 **3 September, 3 PM – 4 PM** **Marks = 20**

Answer all questions.
State all assumptions you make. Keep your answers concise.

1. *Braess Paradox Game.* Figure 1 shows a network that consists of a source S and a destination T, and two intermediate hubs A and B. It is required to travel from S to T. One route is via the hub A and the other route goes via the hub B. Regardless of the number of vehicles on the route, it takes 25 minutes to travel from S to B or from A to T. On the other hand, the travel time from S to A takes time $m/50$ minutes where m is the number of vehicles on that link. Similarly, the travel time from B to T takes time $m/50$ minutes where m is the number of vehicles on that link.

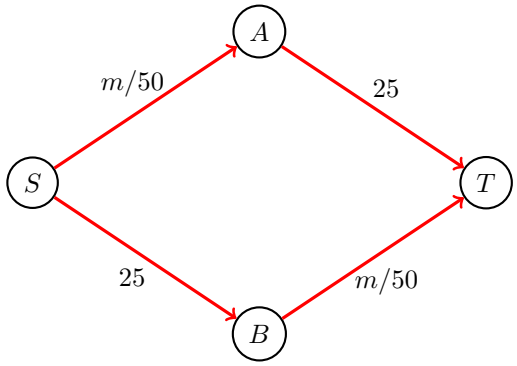


Figure 1

Assume that there are $n = 1000$ vehicles that wish to move from S to T. This means $N = \{1, 2, \ldots, 1000\}$ (and $n = 1000$). The strategy sets are $S_1 = ..., = S_n = \{A, B\}$. Given a strategy profile $(s_1, \ldots, s_n)$, let $n_A(s_1, \ldots, s_n)$ and $n_B(s_1, \ldots, s_n)$ denote the number of vehicles taking the route via A and B respectively. It is easy to note that $n_A(s_1, ..., s_n) + n_B(s_1, ..., s_n) = n$. If utility of a player (in this case vehicle) is defined as the negative of the travel time for that player from S to T, then it is easy to see that

$$u_i(s_1, \ldots, s_n) = \begin{cases} -25 - \frac{n_A(s_1,\ldots,s_n)}{50} & \text{if } s_i = A \\ -25 - \frac{n_B(s_1,\ldots,s_n)}{50} & \text{if } s_i = B \end{cases}$$

(a) Show that the strategy profiles where exactly 500 vehicles take route via A while the rest 500 vehicles take the route via B are pure strategy Nash equilibria.

(b) Suppose a new link from A to B with travel time 0 is introduced as shown in Figure 2. Write down the new strategy sets and the utility functions.

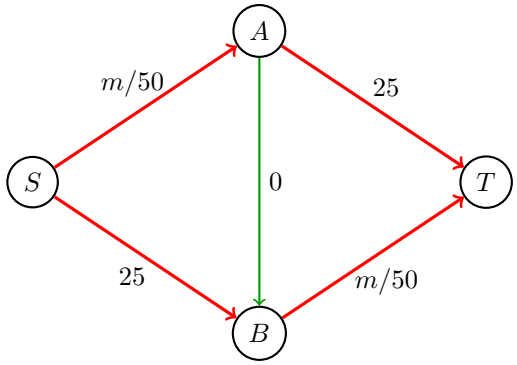


Figure 2

(c) For the extended game with a link, is there a strongly dominant strategy equilibrium? Justify. What is the paradox?

3+2+(2+1) = 8

2. Compute all MSNEs for the following 2-person games.

(a) $d \geq 0$, $S_1 = [d, d+2]$, $S_2 = [d+3, 2d+4]$ and

$$u_1(x, y) = -u_2(x, y) = |x - y| \quad \forall (x, y) \in S_1 \times S_2.$$

(b)

	A	B
A	$(a, 0)$	$(1, 2-a)$
B	$(1, 1)$	$(0, 0)$

where $a > 1$.

3+3 = 6

3. Consider n players with strategy sets $S_1, S_2, \ldots, S_n$ and cost functions $C_1, C_2, \ldots, C_n$. Let $S = S_1 \times S_2 \times \cdots \times S_n$. A *team game* is a game in which all players have the same payoff (cost/utility) functions: $C_1(s) = C_2(s) = \cdots = C_n(s)$ for all strategy profiles $s \in S$. A *dummy game*, the payoff of every player i is independent of its strategy: $C_i(s_i, s_{-i}) = C_i(s_i', s_{-i})$ for all $s_i, s_i' \in S_i$.

Prove that a game with cost functions $C_1, \ldots, C_n$ is a potential game if and only if it is the sum of a team game $C_1^t, \ldots, C_n^t$ and a dummy game $C_1^d, \ldots, C_n^d$. That is, $C_i(s) = C_i^t(s) + C_i^d(s)$ for all $s \in S$ and for every $i \in [n]$.

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