Indian Institute of Technology Kharagpur
Department of Computer Science and Engineering

CS60025 Algorithmic Game Theory **Autumn 2026**

Quiz 3 **12 November, 3:00 PM – 3:20 PM** **Marks: 10 × 1.5 = 15**

Name: _____ **Roll No.:** _____

Please write your answers in the space provided at the end. Do not mark on the questions.

---

1. Consider a three-player sealed-bid second-price auction with known valuations $v_1 = 10$, $v_2 = 7$, and $v_3 = 4$. Ties are broken in favor of the bidder with the smaller index (i.e., Bidder 1 beats Bidder 2, and Bidder 2 beats Bidder 3). Which of the following bid profiles (b_1, b_2, b_3) is a pure strategy Nash equilibrium (PSNE) in which Bidder 3 wins the item?

   A. $(b_1, b_2, b_3) = (3, 2, 11)$
   B. $(b_1, b_2, b_3) = (3, 2, 9)$
   C. $(b_1, b_2, b_3) = (5, 4, 10)$
   D. $(b_1, b_2, b_3) = (6, 1, 11)$

2. In a Pigou-like routing game, a total flow of r units is split between two parallel links (1 and 2) with cost functions $c_1(x) = 1$ and $c_2(x) = x^2$, where x is the flow on the respective link. For which values of r does every equilibrium route all flow on the quadratic link, while the routing that minimizes total cost sends positive flow on both links?

   A. $\frac{1}{\sqrt{3}} < r \leq 1$
   B. $0 < r \leq \frac{1}{\sqrt{3}}$
   C. $r > 1$
   D. $\frac{1}{\sqrt{3}} \leq r < 1$

3. In a selfish load balancing game on identical machines, each task has a positive weight and pays the total load on its chosen machine. Suppose one task moves and strictly reduces its own cost. Writing L_j for the load on machine j, which statement is always true?

   A. The sum of squared loads $\sum_j L_j^2$ strictly decreases after the move, even if the makespan stays the same.
   B. The total load $\sum_j L_j$ strictly decreases after the move.
   C. The number of machines attaining the maximum load strictly decreases after the move.
   D. The makespan $\max_j L_j$ strictly decreases after the move.

4. Four tasks of weights $1, 1, x, x$, where $0 < x \leq 1$, choose between two identical machines. Each task pays the total load on its machine. Consider the assignment in which both unit tasks use the first machine and both weight-x tasks use the second. Among the values of x for which this assignment is a pure strategy Nash equilibrium (PSNE), which one maximizes the ratio of its makespan to the optimal makespan?

   A. $x = \frac{1}{2}$, giving a ratio of $\frac{4}{3}$.
   B. $x = \frac{1}{3}$, giving a ratio of $\frac{3}{2}$.
   C. $x = \frac{2}{3}$, giving a ratio of $\frac{6}{5}$.
   D. $x = 1$, giving a ratio of 1.

5. In a selfish load balancing game on $m \geq 2$ identical machines, let L be the makespan of a PSNE and let OPT be the minimum possible makespan. Suppose one machine has load L and each of the other machines has load at least $L/2$. Using the fact that OPT is at least the average machine load, what is the strongest upper bound this argument gives on L/OPT?

A. $\dfrac{m+1}{m}$

B. 2

C. $\dfrac{2m-1}{m}$

D. $\dfrac{2m}{m+1}$

6. In an extensive form game, player i has m information sets $I_1, I_2, \ldots, I_m$. If there are k_j possible actions available at information set I_j, what is the total number of pure strategies available to player i?

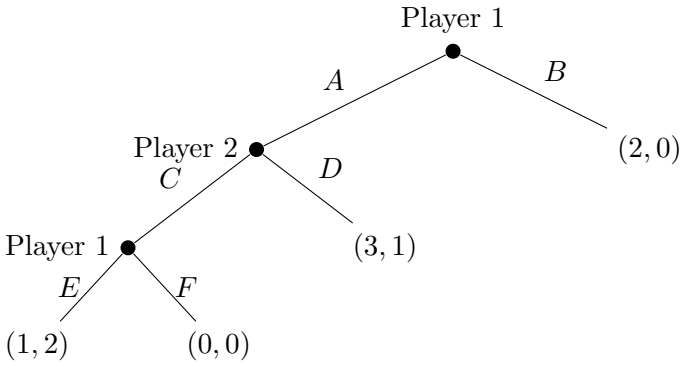
A. $\left(\sum_{j=1}^{m} k_j\right)^m$

B. $\sum_{j=1}^{m} k_j$

C. $\max_{1 \leq j \leq m} k_j$

D. $\prod_{j=1}^{m} k_j$

7. Consider the extensive form game below. Each decision node is a separate information set. Payoffs are written in the order (Player 1, Player 2).



When this game is written in strategic form, which of the following is Player 1's set of pure strategies S_1?

A. $S_1 = \{A, B, E, F\}$

B. $S_1 = \{(A, E), (A, F), (B, E), (B, F)\}$

C. $S_1 = \{(A, C, E), (A, C, F), (A, D), B\}$

D. $S_1 = \{(A, E), (A, F), B\}$

8. Two players each have type H or L. Under a common prior P, Player 1 has type H with probability $1/3$, and Player 2 has type H with probability $1/2$. When Player 1 has type H, her belief is $p_1(\theta_2 = H \mid \theta_1 = H) = 3/4$. For the beliefs to be consistent with P, what must $p_2(\theta_1 = H \mid \theta_2 = H)$ be?

A. $\dfrac{3}{4}$

B. $\dfrac{1}{2}$

C. $\dfrac{1}{3}$

D. $\dfrac{1}{4}$

9. In a first-price sealed-bid auction, two bidders have independent private valuations uniformly distributed on $[0, 1]$. Bidder 2 uses the strategy $s_2(\theta_2) = \theta_2/2$. Bidder 1 knows that her own valuation is $3/4$. Which pair gives her best-response bid and her expected utility from that bid?

   A. Bid $\frac{3}{8}$, expected utility $\frac{9}{32}$.

   B. Bid $\frac{3}{4}$, expected utility 0.

   C. Bid $\frac{1}{2}$, expected utility $\frac{1}{4}$.

   D. Bid $\frac{3}{8}$, expected utility $\frac{3}{8}$.

10. Asha (Player 1) and Ravi (Player 2) simultaneously choose between a music concert (M) and a cricket match (C). Asha has one type, x_1, and Ravi has two possible types, x_2 and y_2. Ravi knows his own type. Asha's beliefs are

$$p_1(x_2 \mid x_1) = \frac{2}{3}, \qquad p_1(y_2 \mid x_1) = \frac{1}{3}.$$

    Asha chooses the row and Ravi chooses the column. Each payoff pair gives Asha's payoff first and Ravi's payoff second.

(x_1, x_2)

	M	C
M	$(5, 3)$	$(1, 0)$
C	$(0, 1)$	$(2, 6)$

(x_1, y_2)

	M	C
M	$(5, 1)$	$(1, 5)$
C	$(0, 3)$	$(2, 0)$

    Which of the following strategy profiles is a PSNE of the corresponding Selten game? The three entries give the actions of types x_1, x_2, and y_2, respectively.

    A. (M, M, C)

    B. (C, C, M)

    C. (M, C, M)

    D. (C, M, C)