Problem Set 4

1. Let $\Gamma = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$ be a game in strategic form. Prove that the joint probability distribution $\sigma \in \Delta(S_1 \times S_2 \times \cdots \times S_n)$ is a Correlated Equilibrium (CE) if and only if the following holds for every player $i \in N$ and every strategy deviation mapping $\delta_i : S_i \to S_i$:

$$\mathbb{E}_{s \sim \sigma}[u_i(s)] \geq \mathbb{E}_{s \sim \sigma}[u_i(\delta_i(s_i), s_{-i})]$$

2. Prove that as the degree p of the cost function on the bottom link of non-linear Pigou's network goes to ∞, the Price of Anarchy (PoA) of Pigou's network tends to ∞ as $\frac{p}{\ln p}$.

3. Prove that in a selfish load balancing game with 3 tasks and 2 identical machines, the Price of Anarchy (PoA) with respect to Pure Strategy Nash Equilibria (PSNE) is 1.

4. In an extensive form game, a certain player has m information sets indexed by $j = 1, 2, \ldots, m$. There are k_j possible actions available at information set j. How many pure strategies does the player have?

5. For the extensive form game shown below, write down the terminal histories, proper sub-histories, information sets, and convert the game to its strategic (normal) form.

