Problem Set 3

1. Compute all correlated equilibrium of the following coordination game.

   ▷ The set of players (N): $\{1, 2\}$

   ▷ The set of strategies: $S_i = \{A, B\}$ for every $i \in [2]$

   ▷ Payoff matrix:

		Player 2	
		A	B
Player 1	A	$(2, 2)$	$(0, 6)$
	B	$(6, 0)$	$(1, 1)$

2. Compute all correlated equilibrium of the following coordination game.

   ▷ The set of players (N): $\{1, 2\}$

   ▷ The set of strategies: $S_i = \{A, B\}$ for every $i \in [2]$

   ▷ Payoff matrix:

		Player 2	
		A	B
Player 1	A	$(2, 2)$	$(0, 0)$
	B	$(0, 0)$	$(1, 1)$

3. Consider the following symmetric 3×3 game:

	X	Y	Z
X	$(0, 0)$	$(1, 2)$	$(2, 1)$
Y	$(2, 1)$	$(0, 0)$	$(1, 2)$
Z	$(1, 2)$	$(2, 1)$	$(0, 0)$

   Let p be the uniform distribution over the three off-diagonal profiles:

$$p(X, Y) = p(Y, Z) = p(Z, X) = \frac{1}{3}, \quad \text{and } p(s_1, s_2) = 0 \text{ otherwise.}$$

   (a) Show that p is a Coarse Correlated Equilibrium (CCE).

   (b) Show that p is **NOT** a Correlated Equilibrium (CE).

4. Let $\Gamma = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$ be a finite normal-form game. Let $\sigma_i \in \Delta(S_i)$ be mixed strategies for each player $i \in N$, and let $\sigma = \prod_{i=1}^{N} \sigma_i$ denote the product probability distribution over strategy profiles $S = \prod_{i=1}^{N} S_i$. Prove that the joint distribution σ is a Correlated Equilibrium (CE) if and only if the mixed strategy profile $(\sigma_i)_{i \in N}$ is a Mixed Strategy Nash Equilibrium (MSNE) of Γ.

5. Let $\alpha \in \Delta(S_1 \times S_2)$ be a Correlated Equilibrium of a 2-player zero-sum matrix game with payoff matrix $A \in \mathbb{R}^{m\times n}$. Prove that the expected payoff to the Row Player, $u_1(\alpha) = \sum_{i,j} \alpha_{i,j} A_{i,j}$, is exactly equal to the minimax value v^* of the game in mixed strategies.

6. Consider T iterations of no-regret dynamics in an n-player normal-form game. Let $p_i^t \in \Delta(S_i)$ denote the mixed strategy played by player i in iteration t, and let $\sigma_t = \prod_{i=1}^n p_i^t$ be the product distribution over outcomes on day t. Prove that an individual day distribution σ_t is an ϵ-Coarse Correlated Equilibrium (ϵ-CCE) if and only if the profile $(p_1^t, p_2^t, \ldots, p_n^t)$ is an ϵ-Nash Equilibrium (with the exact same error parameter ϵ).