

Problem Set 1

1. Each of n people announces a number in the set $\{1, \dots, K\}$. A prize of 1000 rupees is split equally between all the people whose number is closest to $2/3$ of the average number. Formulate this as a normal form game. Find an MSNE for the game. Is it unique? Justify.
2. Prove that the Rock-Paper-Scissors and Matching Pennies games have unique MSNEs.
3. Give an example of a normal form game which does not have any MSNE.
4. There are n departments in IIT Kharagpur. Each department can try to convince the Director to get a certain budget. If h_i is the number of hours of work put in by a department to make the proposal and $c_i = w_i h_i^2$ is cost of this effort to the department, where w_i is a constant. When the effort levels of the departments are $(h_1, h_2, \dots, h_n)$, the total budget that gets allocated to all the departments is:

$$\alpha \sum_{i=1}^n h_i + \beta \prod_{i=1}^n h_i,$$

where α, β are constants. Consider a game where the departments simultaneously and independently decide how many hours to spend on this effort. Show that an SDSE exists if and only if $\beta = 0$. Compute this equilibrium.

5. Consider the following variant of Prisoners' dilemma.

	NC	C
NC	$(-4, -4)$	$(-2, x)$
C	$(-x, -2)$	$(-x, -x)$

Find values of x for which

- (a) the profile (C,C) is a strongly dominant strategy equilibrium.
- (b) the profile (C,C) is a weakly dominant strategy equilibrium but not a strongly dominant strategy equilibrium.
- (c) the profile (C,C) is a not even a weakly dominant strategy equilibrium.

In each case, say whether it is possible to find such an x . Justify your answer in each case.

6. Compute a Nash equilibrium for the 2 person game with $S_1 = [0, 1]$, $S_2 = [3, 4]$ and $u_1(x, y) = -u_2(x, y) = |x - y| \forall (x, y) \in [0, 1] \times [3, 4]$.
7. Compute PSNE for the following 2-player game.

	X	Y	Z
A	$(6, 6)$	$(8, 20)$	$(0, 8)$
B	$(10, 0)$	$(5, 5)$	$(2, 8)$
C	$(8, 0)$	$(20, 0)$	$(4, 4)$

8. Find the mixed strategy Nash equilibria for the following 2-player games.

		H	T
(a)	H	(1, 1)	(0, 1)
	T	(1, 0)	(0, 0)

		A	B
(b)	A	(6, 2)	(0, 0)
	B	(0, 0)	(2, 6)

If all numbers are multiplied by 2, will the equilibria change?

		A	B
(c)	A	(20, 0)	(0, 10)
	B	(0, 90)	(20, 0)

9. An $m \times m$ matrix is called a latin square if each row and each column is a permutation of $(1, 2, \dots, m)$. Compute pure strategy Nash equilibria, if they exist, of a two person game for which a latin square is the payoff matrix.
10. Suppose in a matrix game, the players have 3 strategies each. Which numbers among $0, 1, 2, \dots, 9$ cannot be the total number PSNEs in the matrix game?
11. Construct a two player zero-sum game with $S_1 = \{A, B, C\}$, $S_2 = \{X, Y, Z\}$ with value $= 1/2$ and such that the set of optimal strategies for the row player is exactly the set $\{(\alpha, 1 - \alpha, 0) : \frac{3}{8} \leq \alpha \leq \frac{5}{8}\}$.
12. Army A has a single plane with which it can strike one of three possible targets. Army B has one anti-aircraft gun that can be assigned to one of the targets. The value of target k is v_k , with $v_1 > v_2 > v_3 > 0$. Army A can destroy a target only if the target is undefended and A attacks it. Army A wishes to maximize the expected value of the damage and army B wishes to minimize it. Formulate the situation as a (strictly competitive) game and find its mixed strategy Nash equilibria.
13. Show that the payoffs in Nash equilibrium of a symmetric matrix game (matrix game with symmetric payoff matrix) will be equal to zero for each player.
14. In a normal form game $\Gamma = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$ if a pure strategy $s_i \in S_i$ for some player i is strongly dominated by some mixed strategy $\sigma_i \in \Delta(S_i)$, then in every MSNE of the game, player i chooses the strategy s_i with probability 0.