Indian Institute of Technology Kharagpur
Department of Computer Science and Engineering

CS60025 Algorithmic Game Theory **Autumn 2026**

Mid-Semester Examination **1 October, 9 AM – 11 AM** **Marks = 60**

*Answer **any five** questions. State all assumptions you make. Keep your answers concise.*

1. Consider the game $(N = \{1, 2, \ldots, n\}, (S_i)_{i \in N}, (u_i)_{i \in N})$ where $S_i = \{1, 2, \ldots, n\}$ for all $i \in N$ and
$$u_i(s_1, \ldots, s_n) = \begin{cases} a_{ik} > 0 & \text{if } s_1 = s_2 = \cdots = s_n = k \\ 0 & \text{otherwise} \end{cases}$$
Show that the only pure strategy profiles which are not equilibrium points are those with exactly $n - 1$ of the s_i's equal. [12]

2. Two players in the game of Two-Finger Morra simultaneously put out either one or two fingers. Each player must also announce the number of fingers that he believes his opponent has put out. If neither or both players successfully guess the number of fingers put out by the opponent, the game is a draw. Otherwise, the player who guesses correctly wins from the other player the sum of the fingers put out by the two players. The game is a zero-sum game. Let (i, j) be a strategy of putting out i fingers and guessing the opponent has put out j fingers.
   (a) Write down the game in normal form.
   (b) Are there any pure strategy Nash equilibria?
   (c) Verify whether or not $x^* = \left(\frac{5}{12}, \frac{4}{12}, \frac{3}{12}, 0\right)$, $y^* = \left(\frac{5}{12}, \frac{4}{12}, \frac{3}{12}, 0\right)$ (corresponding to strategies $(1,1), (1,2), (2,1), (2,2)$) forms a mixed strategy Nash equilibrium for the game.
   (d) Consider a variant where
   - if the sum is even, Player 1 wins the total sum of fingers
   - if the sum is odd, Player 2 wins the total sum of fingers

   Write down the game in normal form. Find MSNE for this game.

   [3+2+3+4 = 12]

3. Consider a bimatrix game (A, B) (row/column player has m/n strategies respectively, $A, B \in \mathbb{R}^{m \times n}$). We call $(\vec{x}^*, \vec{y}^*)$ an ϵ-approximate Nash equilibrium if $(\vec{x}^*)^T A \vec{y}^* \geq \vec{x}^T A \vec{y}^* - \epsilon$ for all row mixed strategies $(\vec{x}^*)^T A \vec{y}^* \geq \vec{x}^* B \vec{y} - \epsilon$ for all column mixed strategies $\vec{y}$. Prove that there is a $\frac{1}{2}$-approximate Nash equilibrium in which one player plays a pure strategy and the other randomizes uniformly between two strategies. [12]

4. Consider the multiplicative weights algorithm. Suppose we modify the weight update step as $w_t(a) = w_{t-1}(a)(1 + \epsilon \pi_t(a))$ (instead of $w_t(a) = w_{t-1}(a)(1 + \epsilon)^{\pi_t(a)}$) for every $a \in A$. Does the modified algorithm have regret $O(\sqrt{\log n}/\sqrt{T})$? Justify. [12]

5. Prove that if a game admits two potential functions Φ_1 and Φ_2 then Φ_1 and Φ_2 differ by a constant. That is, for some $c \in \mathbb{R}$, $\Phi_1 = \Phi_2 + c$ for every outcome s of the game. [12]

6. Suppose a finite cost minimization game admits a function Φ with the property that, for every strategy profile $\mathbf{s}$, every player i, and every deviation s_i' with $C_i(s_i', \mathbf{s}_{-i}) < C_i(\mathbf{s})$, we have $\Phi(s_i', \mathbf{s}_{-i}) < \Phi(\mathbf{s})$. Is this enough to conclude that the game has at least one pure strategy Nash equilibrium? [12]

7. (a) Define correlated equilibrium (CE) and ϵ-CE for a game $\Gamma = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$ with $N = \{1, 2, \ldots, n\}$.

   (b) Prove that the distribution $\sigma \in \Delta(S_1 \times S_2 \times \cdots \times S_n)$ is a CE if and only if the following holds for every $i \in N$ and every $\delta_i : S_i \to S_i$.

$$\mathbf{E}_{s \sim \sigma}[u_i(s)] \geq \mathbf{E}_{s \sim \sigma}[u_i(\delta_i(s_i), s_{-i})].$$

   (c) Suppose after T iterations of no-swap-regret dynamics, every player of a cost-minimization game (of n players) has swap regret at most ϵ for each of its switching functions. Let $\sigma_t = \prod_{i=1}^{n} p_i^t$ denote the outcome distribution at time t and $\sigma = \frac{1}{T} \sum_{t=1}^{T} \sigma_t$ the time-averaged history of these distributions. Show that σ is an ϵ-approximate correlated equilibrium. [Here, every player i independently chooses a mixed strategy p_i^t according to a no-swap-regret algorithm.]

2+8+2 = 12