

Please write your answers in the space provided. Do not mark in the question itself.

- Which of the following polynomials is irreducible in $\mathbb{F}_2[x]$?
 - $x^4 + x^3 + x + 1$
 - $x^4 + x^2 + x + 1$
 - $x^4 + x^2 + 1$
 - $x^4 + x^3 + 1$
- The gcd of a polynomial $f(x) \in \mathbb{F}_q[x]$ with $x^{q^2} - x \in \mathbb{F}_q[x]$ tells us whether $f(x)$
 - has quadratic and linear factors**
 - completely factors into linear factors over \mathbb{F}_q .
 - has any roots in \mathbb{F}_q
 - has linear factors
- Let $\mathbb{F}_{16} = \mathbb{F}_{2^4} = \mathbb{F}_2(\theta)$ where θ is a root of the polynomial $x^4 + x^3 + x^2 + x + 1 \in \mathbb{F}_2[x]$. The element $\theta^2 + 1$ is
 - primitive but not normal
 - primitive normal
 - normal but not primitive**
 - neither primitive nor normal.
- p -th power exponentiations in \mathbb{F}_{p^n} is most efficient in which of the following representations
 - polynomial basis
 - discrete log representation
 - normal basis**
 - arbitrary basis
- Running time of the algorithm for testing irreducibility of $f(x) \in \mathbb{F}_q[x]$ with $\deg f = d$ is
 - $O(d^2 \log^3 q)$
 - $O(d^3 \log^2 q)$
 - $O(d^3 \log^3 q)$**
 - $O(d^2 \log^2 q)$
- Which of the following statements is incorrect?
 - Every Euler pseudoprime to base a is a strong pseudoprime to base a .
 - Every Euler pseudoprime to base a is a Fermat pseudoprime to base a .**

- C. Every Fermat pseudoprime to base a is a strong/Miller-Rabin pseudoprime to base a .
- D. Every Fermat pseudoprime to base a is an Euler pseudoprime to base a .
7. Let n be an odd composite integer. Which of the following is correct?
- A. n is not an Euler pseudoprime to at least half the bases in \mathbb{Z}_n^* .**
- B. n is a Fermat pseudoprime to all bases in \mathbb{Z}_n^* .
- C. n is not an Fermat pseudoprime to at least half the bases in \mathbb{Z}_n^* .
- D. There is at least one base in \mathbb{Z}_n^* to which n is not a Fermat pseudoprime.
8. Let $\{F_m\}_{m \geq 0}$ denote the Fibonacci sequence. Which of the following statements about the Fibonacci test is incorrect?
- A. It is a deterministic test
- B. The test is based on the fact that n is prime iff $F_{n - (\frac{3}{n})} \equiv 0 \pmod n$**
- C. The test is based on the fact that n is prime iff $F_{n - (\frac{5}{n})} \equiv 0 \pmod n$
- D. Failure depends on distribution of Fibonacci pseudoprimes
9. Which of the following is incorrect?
- A. $F_{2k+2} = F_{k+1}(F_{k+1} + 2F_k)$
- B. $F_{2k+1} = F_{k+1}^2 + F_k^2$
- C. $F_{2k} = F_k(2F_{k+1} - F_k)$
- D. $F_{2k} = F_{k+1}^2 + F_k^2$**
10. Running time of Miller-Rabin primality test (on input n) is
- A. $O(\log^3 n)$**
- B. $O(n \log n)$
- C. $O(n)$
- D. $O(\log^4 n)$