Problem Set 1

1. Each of n people announces a number in the set $\{1, \dots, K\}$. A prize of 1000 rupees is split equally between all the people whose number is closest to 2/3 of the average number. Formulate this as a normal form game. Find an MSNE for the game. Is it unique? Justfy.

2. Prove that the Rock-Paper-Scissors and Matching Pennies games have unique MSNEs.

3. Give an example of a normal form game which does not have any MSNE.

4. There are n departments in IIT Kharagpur. Each department can try to convince the Director to get a certain budget. If h_i is the number of hours of work put in by a department to make the proposal and $c_i = w_i h_i^2$ is cost of this effort to the department, where w_i is a constant. When the effort levels of the departments are $(h_1, h_2, \dots, h_n)$, the total budget that gets allocated to all the departments is:

$$\alpha \sum_{i=1}^{n} h_i + \beta \prod_{i=1}^{n} h_i,$$

where α, β are constants. Consider a game where the departments simultaneously and independently decide how many hours to spend on this effort. Show that an SDSE exists if and only if $\beta = 0$. Compute this equilibrium.

5. Consider the following variant of Prisoners' dilemma.

	NC	C
NC	$(-4, -4)$	$(-2, x)$
C	$(-x, -2)$	$(-x, -x)$

Find values of x for which

(a) the profile (C,C) is a strongly dominant strategy equilibrium.

(b) the profile (C,C) is a weakly dominant strategy equilibrium but not a strongly dominant strategy equilibrium.

(c) the profile (C,C) is a not even a weakly dominant strategy equilibrium.

In each case, say whether it is possible to find such an x. Justify your answer in each case.

6. Compute a Nash equilibrium for the 2 person game with $S_1 = [0, 1]$, $S_2 = [3, 4]$ and

$$u_1(x, y) = -u_2(x, y) = |x - y| \quad \forall (x, y) \in [0, 1] \times [3, 4].$$

7. Compute PSNE for the following 2-player game.

	X	Y	Z
A	$(6,6)$	$(8,20)$	$(0,8)$
B	$(10,0)$	$(5,5)$	$(2,8)$
C	$(8,0)$	$(20,0)$	$(4,4)$

8. Find the mixed strategy Nash equilibria for the following 2-player games.

   (a)

	H	T
H	$(1,1)$	$(0,1)$
T	$(1,0)$	$(0,0)$

   (b)

	A	B
A	$(6,2)$	$(0,0)$
B	$(0,0)$	$(2,6)$

   If all numbers are multiplied by 2, will the equilibria change?

   (c)

	A	B
A	$(20,0)$	$(0,10)$
B	$(0,90)$	$(20,0)$

9. An $m \times m$ matrix is called a latin square if each row and each column is a permutation of $(1, 2, \ldots, m)$. Compute pure strategy Nash equilibria, if they exist, of a two person game for which a latin square is the payoff matrix.

10. Suppose in a matrix game, the players have 3 strategies each. Which numbers among $0, 1, 2, \ldots, 9$ cannot be the total number PSNEs in the matrix game?

11. Construct a two player zero-sum game with $S_1 = \{A, B, C\}$, $S_2 = \{X, Y, Z\}$ with value $= 1/2$ and such that the set of optimal strategies for the row player is exactly the set

$$\left\{(\alpha, 1-\alpha, 0) \,:\, \frac{3}{8} \leq \alpha \leq \frac{5}{8}\right\}.$$

12. Army A has a single plane with which it can strike one of three possible targets. Army B has one anti-aircraft gun that can be assigned to one of the targets. The value of target k is v_k, with $v_1 > v_2 > v_3 > 0$. Army A can destroy a target only if the target is undefended and A attacks it. Army A wishes to maximize the expected value of the damage and army B wishes to minimize it. Formulate the situation as a (strictly competitive) game and find its mixed strategy Nash equilibria.

13. Show that the payoffs in Nash equilibrium of a symmetric matrix game (matrix game with symmetric payoff matrix) will be equal to zero for each player.