Problem Set 3

1. Let $\Gamma = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$ be a game in strategic/normal form. Let $\sigma_i \in \Delta(S_i)$ be mixed strategies for the players and let $\sigma = \prod_{i=1}^{N} \sigma_i$. Prove that σ is a CE if and only if $(\sigma_i)_{i \in N}$ is an MSNE.

2. Let $\Gamma = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$ be a game in strategic form. Prove that the distribution $\sigma \in \Delta(S_1 \times S_2 \times \cdots \times S_n)$ is a CE if and only if the following holds for every $i \in N$ and every $\delta_i : S_i \to S_i$.
$$\mathbf{E}_{s \sim \sigma}[u_i(s)] \geq \mathbf{E}_{s \sim \sigma}[u_i(\delta_i(s_i), s_{-i})]$$

3. Let α be a correlated equilibrium of a matrix game. Prove that $u_1(\alpha)$ (the utility of the row player) is equal to the value of the game in mixed strategies.

4. Compute all correlated equilibria of the following coordination games with $N = \{1, 2\}$, $S_1 = S_2 = \{A, B\}$.

(a)

	A	B
A	$(2, 2)$	$(0, 6)$
B	$(6, 0)$	$(1, 1)$

(b)

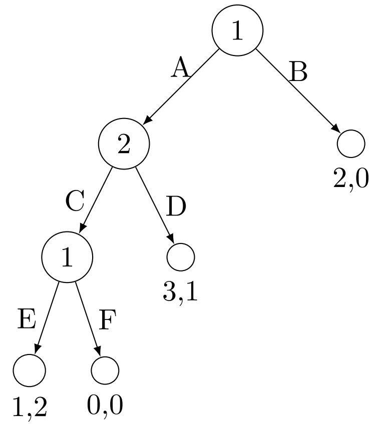
	A	B
A	$(2, 2)$	$(0, 0)$
B	$(0, 0)$	$(1, 1)$

5. Prove that as the degree p of the cost function in the bottom link of Pigou's network goes to ∞, the price of anarchy of Pigou's network tends to ∞ as $\frac{p}{\ln p}$.

6. Prove that in a selfish load balancing game with 3 tasks and 2 identical machines, the PoA with respect to PSNE is 1.

7. In an extensive form game, a certain player has m information sets indexed by $j = 1, 2, \ldots, m$. There are k_j possible actions for information set j. How many strategies does the player have?

8. For the game shown below write down the terminal histories, proper subhistories, information sets and convert to strategic form game.



9. Consider T iterations of no-regret dynamics in an n-player game. Let p_i^t denote the mixed strategy played by player i in iteration t, and $\sigma_t = \prod_{i=1}^n p_i^t$ the corresponding distribution over outcomes on day t. In lecture we proved that the time-averaged joint distribution $\frac{1}{T}\sum_{t=1}^T \sigma_t$ is an approximate coarse correlated equilibrium, but did not claim anything about any individual distribution σ_t. Prove that an individual distribution σ_t is an approximate coarse correlated equilibrium if and only if it is an approximate Nash equilibrium (with the same error term).