Problem Set 2

1. Consider a 3 person game with $S_1 = S_2 = S_3 = \{1, 2, 3, 4\}$. If $u(x, y, z) = x + y + z + 4i$ for each $i = 1, 2, 3$, show that the game has a unique Nash equilibrium.

2. Compute all Nash equilibria for the following game for each $a \in (1, \infty)$.

	A	B
A	$(a, 0)$	$(1, 2 - a)$
B	$(1, 1)$	$(0, 0)$

3. Describe a game with a PSNE and an initial strategy profile starting from which best-response dynamics cycles forever.

4. Provide an example of a network congestion game where best-response dynamics takes an exponential number of iterations to converge to a PSNE.

5. In the 2SAT problem, an instance is a boolean formula in conjunctive normal form where every clause has exactly 2 literals (e.g. $(x_1 \vee x_3) \wedge (\neg x_2 \vee x_3), (\neg x_2 \vee x_4) \wedge (x_2 \vee \neg x_3) \wedge (x_1 \vee \neg x_4)$). In Local-Weighted-Max-2SAT problem, we are given a set of 2SAT clauses each having a weight. An assignment of the variables is said to satisfy a clause if and only if it makes at least one of its literal true. The goal is to find an assignment which is locally optimal – by changing the assignment of any one variable, it is not possible to increase the sum of weights of the clauses satisfied. Prove that Local-Weighted-Max-2SAT is **PLS**-complete.