Game-Theoretic Distributed Virtual Energy Cloud Topology Control for Mobile Smart Grid

Ayan Mondal
Student Member, IEEE
School of Information Technology
Indian Institute of Technology Kharagpur
Kharagpur-721302, India
Email: ayanmondal@sit.iitkgp.ernet.in

Sudip Misra
Senior Member, IEEE
School of Information Technology
Indian Institute of Technology Kharagpur
Kharagpur-721302, India
Email: smisra@sit.iitkgp.ernet.in

***Abstract*—In this paper, the problem of energy distribution using virtual energy-cloud to the plug-in hybrid electric vehicles (PHEVs) is studied as a single leader multiple follower non-cooperative Stackelberg game. In this game, the energy-cloud service provider acts as the leader, and decides the price to be paid by each PHEV according to its usage. On the other hand, the PHEVs act as the followers, and need to decide the amount of energy to be consumed based on their requirements. Using variational inequality, it is shown that the proposed scheme, *virtual energy cloud topology control* (VELD), has a generalized Nash equilibrium, which is also socially optimal. The proposed scheme, VELD, which enables the energy-cloud service provider and the PHEVs to reach the equilibrium state, is evaluated theoretically as well as through simulations. Using the proposed scheme, VELD, the PHEVs consume up to $47.49-52.96\%$ higher amount of energy, while paying 5.52% less per unit energy, which, in turn, increases the utilization of the generated energy by the micro-grids.**

Keywords—Virtual Energy Cloud, Topology Control, Plug-in Hybrid Electric Vehicle, Smart Grid, Stackelberg Game, Non-cooperative Game.

I. INTRODUCTION

In order to achieve high quality of energy service, the electrical grids need to be modernized by integrating advanced information and communication techniques, and it is named as *smart grid* [1], [2]. Smart grid is conceptualized as a combination of electrical and communication networks. With the bidirectional energy exchange facilities, a modernized smart grid is capable of delivering energy more efficiently and reliably to customers than the traditional electrical grid. A smart grid also integrates several advanced techniques such as advanced metering infrastructure (AMI), automatic meter reading, energy management systems (EMS), and plug-in hybrid electrical vehicles (PHEVs) [3]. In smart grid, the traditional large electrical grid is divided into several small geographical areas, which are served by the micro-grids. The micro-grids exchange their excess amount of energy to the other micro-grids with deficiency of energy or to the substation. On the other hand, the PHEVs request energy to the micro-grids flexibly, based on their requirements. Therefore, a distributed energy topology control mechanism is needed to ensure the quality of energy service in mobile smart grid.

The micro-grids generate energy typically based on the renewable energy resources. Therefore, the amount of generated energy is not fixed for different time slots. If a micro-grid has excess amount of generated energy, it supplies that excess amount either to the other micro-grids having demand of energy, or to the main grid through the substations. In those cases, the loss of energy through the transmission line increases. On the other hand, if a set of PHEVs requests a certain amount of energy to any micro-grid, which is higher than the amount of generated energy by it, the micro-grid charges the PHEVs with a very high price. If the PHEVs are not wiling to pay very high price, they wait for a certain duration of time to get the energy service. Existing pieces of literature (eg. [4]) on energy distribution consider that in on-peak hours, the amount of energy requested to the micro-grids is high, whereas same to the off-peak hour is low. Therefore, the PHEVs have to keep in mind the time they are requesting energy to the micro-grids. Moreover, using the existing schemes, either the PHEVs have to pay high, or have to wait for longer time. Therefore, we propose a *virtual energy cloud service* scheme. In this scheme, the energy-cloud service provider [5] is responsible for providing the energy service to the connected PHEVs with better quality of service (QoS). On the other hand, the PHEVs do not wait for a longer duration to get their energy services, and they also pay as per their usages. Therefore, using the energy-cloud service provider, the PHEVs consume energy as per their requirement, while ensuring high QoS. On the other hand, the micro-grids having excess energy get the revenue by supplying the excess amount of energy.

In this paper, we propose a non-cooperative game theoretic scheme, named VELD, for *virtual energy cloud topology control*. We use a single leader multiple follower Stackelberg game to decide the amount of energy to be consumed by each PHEV to fulfill its energy requirement, while expending less money. On the other hand, the energy-cloud service provider [5] decides its strategy, i.e., price per unit energy, to earn revenue by selling the generated amount of energy by the micro-grids connected to the energy-cloud service provider. We summarize the contributions of this paper as follows:

(i) We propose two algorithms for *virtual energy cloud topology control game* to evaluate the real-time energy consumption of the PHEVs in the presence of an energy-cloud service provider. Each follower, i.e., PHEV, decides its strategy, i.e., the amount of energy to be consumed, based on the local information. On the other hand, the leader, i.e., the energy-cloud service provider, decides the price per unit energy based on the amount of energy requested by the PHEVs.

(ii) A single leader multiple follower Stackelberg game is used to evaluate the optimal strategies of the PHEVs using a non-cooperative game theoretic approach, and the optimum price per unit energy is decided by the energy-cloud service provider.

The remainder of the paper is organized as follows. Section II summarizes the related work in the area of virtual cloud-based energy management in smart grid. The system model is discussed in Section III. The proposed scheme, VELD, is described in Section IV. Section V depicts the simulation setup and performance evaluation of the proposed scheme, VELD, considering different performance metrics. Finally, Section VI concludes the paper by citing directions for future work.

II. Related Work

In the last few years, lots of research work on cloud applications on smart grid, viz., [1]–[16], have been done. Some of the existing literature are discussed in this Section. Kim *et al.* [10] proposed a cloud-based demand response architecture using a master-slave architecture. Two cloud-based demand response models are proposed such as data-centric communication, and topic-based group communication. However, the authors did not consider any energy distribution approach using cloud. Li *et al.* [12] proposed a scheduling approach of the submitted jobs to the cloud service provider, and the service provider schedules the jobs based on their priorities, and energy requirements. However, the authors did not focus on the energy distribution approach using cloud. Rajeev and Ashok [15] proposed a framework for integrating cloud computing applications for micro-grid management in different modules — infrastructure, power management, and service. They proposed a scheduling approach, where different operators publish their service descriptions using the service modules. Some of the existing works done on smart grid without the cloud infrastructure are also discussed in this Section. Such and Hill [16] proposed that efficient and economic operation of an electric energy distribution system can be improved with the implantation of wind energy and storage. Tushar *et al.* [4] proposed a charging scheme of the PHEVs without any cloud infrastructure. Misra *et al.* [3] proposed a pricing scheme in mobile smart grid. However, they did not consider any energy-cloud infrastructure.

In contrast to the existing work, a game theoretic distributed virtual energy cloud topology control is proposed for PHEVs functioning in a mobile smart grid environment. We use a single leader multiple follower Stackelberg game theoretic approach to develop an optimal solution of energy distribution using the energy-cloud infrastructure in mobile smart grid.

III. System Model

We consider an energy distribution topology consisting two-layered architectures — *mobile macro-grid*, and *virtual energy-cloud*. Mobile macro-grid architecture consists of multiple mobile plug-in hybrid electric vehicles (PHEVs), and a single energy-cloud service provider [5]. On the other hand, a virtual energy-cloud architecture consists of a single energy-cloud service provider, and multiple micro-grids. The PHEVs demand the required amount of energy to the energy-cloud service provider. Hence, based on the mobility pattern of the PHEVs, the energy-cloud service provider maps the mobile PHEVs to the suitable energy generation units, i.e., micro-grids, such that the loss of energy through the transmission line, and energy service delay are minimum. In addition, if a PHEV travels long distance for an energy charging station (ECS), which is defined in Definition 1, the residual energy of the PHEV is reduced and the delay in getting the energy service also increases. Therefore, the energy requirement of the PHEV increases, i.e., the PHEV has to consume higher amount of energy to charge its battery fully. The schematic diagram of the energy distribution topology is shown in Figure 1.

Definition 1. *An energy charging station (ECS) is used as an energy exchange point between the PHEVs and the micro-grids using a virtual energy-cloud. We consider that in a small geographical area, there are multiple ECSs such that the PHEVs within that region get prompt service as per their requirements.*

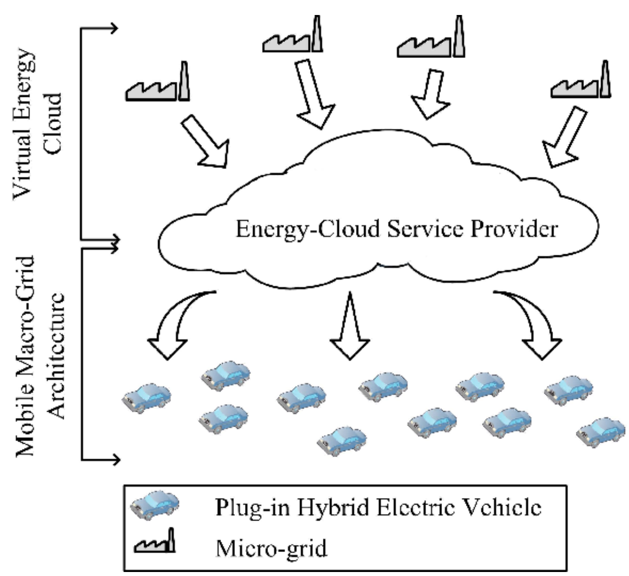

Fig. 1: Schematic diagram of energy distribution topology

A. Mobile Macro-Grid Architecture

We consider that at time instant $t \in [0, \mathbb{T}]$, where $\mathbb{T}$ is the number of time instants in a day, the energy-cloud service provider supplies energy to each PHEV $n \in \mathcal{N}(t)$, where $\mathcal{N}$ is the total number of available PHEVs in mobile smart grid at time instant t. We assume that at time instant t, each PHEV $n \in \mathcal{N}(t)$ demands $d_n(t)$ amount of energy to the energy-cloud service provider to fulfill its energy requirement. On the other hand, the energy-cloud service provider charges each PHEV $n \in \mathcal{N}(\cdot)$ based on the energy consumption profile. Hence, we consider that the energy-cloud service provider uses a linear pricing mechanism for deciding on the amount of price to be paid by each PHEV $n \in \mathcal{N}(\cdot)$, individually. We discuss about the pricing scheme of the energy-cloud service provider in Section III-A1.

1) Pricing Scheme of the Energy-Cloud Service Provider: The energy-cloud service provider decides the price per unit energy, i.e., $p_n(\cdot)$, to be paid by each PHEV $n \in \mathcal{N}(\cdot)$ based on the amount of energy request by PHEV $n \in \mathcal{N}(\cdot)$, i.e., $d_n(\cdot)$. As the energy cloud service provider tries to

ReqMsgType	PHEV_ID	ReqEnergy	FinalSelFlag
1 byte	4 byte	2 byte	1 byte

(a) Request Message

AckMsgType	CSP_ID	Price	AckFlag
1 byte	4 byte	2 byte	1 byte

(b) Acknowledgement Message

Fig. 2: Message format in proposed VELD scheme

ReqMsgType	CSP_ID	ReqEnergy	AckFlag
1 byte	4 byte	2 byte	1 byte

(a) Request Message

AckMsgType	MicroGrid_ID	SelectionFlag
1 byte	4 byte	1 byte

(b) Acknowledgement Message

Fig. 3: Message format in virtual energy-cloud game

maximize its revenue by considering a trade-off between the price per unit energy and the amount of energy supplied, while maintaining its minimum revenue. Therefore, the energy-cloud service provider uses convex pricing function, i.e., $\mathscr{P}_n(\cdot)$, for its pricing scheme, as follows:

$$\begin{aligned} \mathscr{P}_n(\cdot) &= p_n(\cdot)d_n(\cdot), \quad \forall n \in \mathcal{N}(\cdot) \\ &= \left[c^{avg} + \tan^{-1}\left(e^{\sum d_n(\cdot)}\right)\right] d_n(\cdot) \end{aligned} \tag{1}$$

where c^{avg} is the average energy generation cost per unit energy of the micro-grids connected with energy-cloud service provider. We define the average energy generation cost of the micro-grids, i.e., c^{avg}, mathematically, as follows:

$$c^{avg} = \frac{\sum\limits_{m \in \mathcal{M}} c_m}{|\mathcal{M}|} \tag{2}$$

where $\mathcal{M}$ is the available micro-grids connected with the energy-cloud service provider, and c_m is the energy generation cost per unit energy of each micro-grid $m \in \mathcal{M}$.

B. Virtual Energy-Cloud Architecture

The energy-cloud service provider provides the users *Energy-as-a-Service* (EaaS) defined in Definition 2. In EaaS, the users, i.e., the PHEVs, request the energy-cloud service provider to fulfill their energy demands. Based on the demand, the energy-cloud service provider distributes the energy request to the available micro-grids using a load balancing algorithm. Therefore, the energy-cloud service provider enables an infrastructure to provide the energy service to the available PHEVs, i.e., it provides the infrastructure for enegry service. The energy-cloud service provider serves energy to the PHEVs based on the demanded energy by the PHEVs on a real-time basis.

Definition 2. *Using Energy as a Service (EaaS), the energy-cloud service provider distributes energy to the PHEVs from the micro-grids. The PHEVs communicate with the micro-grids only through the cloud interface, and the PHEVs do not concern about the availability of energy, as the responsibility of providing energy service solely depends on the energy-cloud service provider. On the other hand, the PHEVs pay depending on the pay-per-use mechanism, i.e., each PHEV has to pay based on the amount of consumed energy decided using the pricing scheme in Section III-A1.*

C. Mobility Model for Cloud-based Mobile Smart Grid

We consider that the mobile PHEVs follow the Gauss-Markov mobility model. According to the mobility model, each PHEV updates its location after traveling a certain distance. The velocity and the position of each PHEV are considered as the correlated functions which are time dependent in nature. Therefore, the velocity and the position of a PHEV at time instant $t \in \mathbb{T}$ depend on the velocity and the position of that PHEV at time instant $(t-1)$. We assume that the PHEVs are mobile in a two-dimensional plane, i.e., 2D plane. The Gauss-Markov mobility model is represented as in [17]:

$$\vec{\nu}_n(t) = \nu_n^x(t)\vec{i} + \nu_n^y(t)\vec{j}, \quad \forall n \in \mathcal{N}(\cdot) \tag{3}$$

where $\vec{i}$ and $\vec{j}$ are the unit vector, $\vec{\nu}_n(\cdot)$ is the velocity vector of PHEV n, and $\nu_n^x(\cdot)$ and $\nu_n^y(\cdot)$ are the velocity components of PHEV $n \in \mathcal{N}(\cdot)$ in X-direction and Y-direction, respectively. We define the velocity components in X-direction and Y-direction, i.e., $\nu_n^x(\cdot)$ and $\nu_n^y(\cdot)$, are as follows:

$$\nu_n^x(t) = \beta\nu_n^x(t-1) + (1-\beta)\gamma^x + \theta(t-1)\sigma^x\sqrt{1-\beta^2} \tag{4}$$

$$\nu_n^y(t) = \beta\nu_n^y(t-1) + (1-\beta)\gamma^y + \theta(t-1)\sigma^y\sqrt{1-\beta^2} \tag{5}$$

where β is the variance over time; γ^x and γ^y are the mean velocity in X-direction and Y-direction, respectively; σ^x and σ^y are the standard deviation of velocity components in X and Y-direction, respectively; and $\theta(\cdot)$ is the time independent uncorrelated Gaussian process with zero-mean with unit variance. In the virtual energy cloud topology control (VELD) scheme, we consider that the variance over time, i.e., the value of β, is within zero and one. Mathematically,

$$0 \leq \beta \leq 1 \tag{6}$$

Hence, we define the magnitude and angle of direction of the velocity of each mobile PHEV $n \in \mathcal{N}(\cdot)$ as given below:

$$|\vec{\nu}_n(\cdot)| = \sqrt{[\nu_n^x(\cdot)]^2 + [\nu_n^y(\cdot)]^2} \tag{7}$$

$$\alpha_n(\cdot) = \tan^{-1}\left(\frac{\nu_n^y(\cdot)}{\nu_n^x(\cdot)}\right) \tag{8}$$

where $|\vec{\nu}_n(\cdot)|$ and $\alpha_n(\cdot)$ are the magnitude and the angle of direction of the velocity of each PHEV $n \in \mathcal{N}(\cdot)$.

D. Communication Model for Cloud-based Mobile Smart Grid

We consider that in EaaS, the energy-cloud service provider communicates with the plug-in hybrid electric vehicles (PHEVs) using wireless mesh network (WMN). We use IEEE 802.11b protocol for the communication purpose. Initially, each PHEV requests the energy-cloud service provider to supply the required amount of energy by sending a request message, as shown in Figure 2(a). Thereafter, the energy-cloud service provider sends an acknowledgment message to the PHEV, as shown in Figure 2(b). Each acknowledgment message is unicasted by the energy-cloud service provider. After getting conformation message, i.e., `FinalSelFlag` in Request message is set, the energy-cloud service provider (ECSP) sends the Request messages, as shown in Figure 3(a), to the connected micro-grids. On getting the request messages, the micro-grids cooperates within themselves, and send an acknowledgment message, as shown in Figure 3(b), while ensuring that each micro-grids connected with the energy-cloud service provider gets the same payoff.

IV. Proposed Virtual Energy Cloud Topology Control Game

A. Game Formulation

To study the interaction between the PHEVs, and the energy-cloud service provider, i.e., for EaaS, we use a single leader multiple follower game theoretic approach in *virtual energy cloud topology control* (VELD) scheme. In this game, the energy-cloud service provider acts as leader, and decides the price per unit energy based on the amount of energy to be consumed by the PHEVs $\mathcal{N}(\cdot)$. On the other hand, the PHEVs act as the followers. Each PHEV $n \in \mathcal{N}(\cdot)$ decides on the amount of energy to be consumed to fulfill its energy requirement. We consider that each PHEV $n \in \mathcal{N}(\cdot)$ decides to consume $d_n(\cdot)$ amount of energy from the energy-cloud service provider. Therefore, the total energy requested, i.e., $\mathcal{D}(\cdot)$, to energy-cloud service provider is defined as follows:

$$\mathcal{D}(\cdot) = \sum_{n=1}^{n \in \mathcal{N}(\cdot)} d_n(\cdot) \tag{9}$$

Based on the total amount of energy requested by the PHEVs, i.e., $\mathcal{D}(\cdot)$, the energy-cloud service provider decides the price per unit energy, i.e., $\mathcal{P}(\cdot)$, using a convex function defined as follows:

$$\mathcal{P}(\cdot) = c^{avg} + \tan^{-1}\left(e^{\mathcal{D}(\cdot)}\right) \tag{10}$$

Hence, from Equation (1), we conclude that the price per unit energy paid by each PHEV $n \in \mathcal{N}(\cdot)$, i.e., $p_n(\cdot)$, is same for the PHEVs connected with the energy-cloud service provider. Mathematically,

$$\mathcal{P}(\cdot) \triangleq p_1(\cdot) \triangleq \cdots \triangleq p_n(\cdot) \triangleq \cdots \triangleq p_{|\mathcal{N}(\cdot)|}(\cdot) \tag{11}$$

The price per unit energy paid by each PHEV n, $p_n(\cdot)$, is not only dependent on the amount of energy requested by PHEV n, $d_n(\cdot)$, but also dependent on the amount of energy requested by the PHEVs other than PHEV n, i.e., $\boldsymbol{d}_{-n}$, where $\boldsymbol{d}_{-n} = \{d_1, d_2, \cdots, d_{n-1}, d_{n+1}, \cdots, d_{|\mathcal{D}(\cdot)|}\}$. Hence, each PHEV $n \in \mathcal{N}(\cdot)$ decides the amount of energy to be consumed with non-cooperation. We define the components of the mobile macro-grid game as follows:

(i) Each PHEV $n \in \mathcal{N}(\cdot)$ acts as a follower, and needs to decide the optimum value of the amount of energy to be consumed, i.e., $d_n(\cdot)$.

(ii) The utility function of each PHEV n, i.e., $\phi_n(\cdot)$, needs to be maximized while depending on the amount of energy to be consumed by PHEV n, i.e., $d_n(\cdot)$, and the price per unit energy, $\mathcal{P}(\cdot)$, decided by the energy-cloud service provider.

(iii) The price per unit energy, $\mathcal{P}(\cdot)$, depends on the total amount of requested energy by the PHEVs, as shown in Equation (10).

(iv) The utility function of the energy-cloud service provider, i.e., $\varphi(\cdot)$, depends on the decided price per unit energy, i.e., $\mathcal{P}(\cdot)$, and the amount of requested energy by each PHEV n, i.e., $d_n(\cdot)$, where $\forall n \in \mathcal{N}(\cdot)$.

a) Utility function of a PHEV: The utility function of PHEV $n \in \mathcal{N}(\cdot)$, i.e., $\phi_n(\cdot)$, is defined as a concave function, and signifies the satisfaction level of PHEV n by consuming $d_n(\cdot)$ amount of energy with a optimum price per unit energy, $p_n(\cdot)$. The satisfaction level of each PHEV n is defined in Definition 3. For requesting $d_n(\cdot)$ amount of energy to the energy-cloud service provider, the net utility of PHEV n, i.e., $\phi_n(\cdot)$, is expressed as the difference between the revenue function of PHEV n, i.e., $\mathscr{R}_n(\cdot)$, and the cost function of PHEV n, i.e., $\mathscr{C}_n(\cdot)$. Mathematically,

$$\phi_n(\cdot) = \mathscr{R}_n(\cdot) - \mathscr{C}_n(\cdot), \quad \forall n \in \mathcal{N}(\cdot) \tag{12}$$

Definition 3. *The satisfaction level of PHEV $n \in \mathcal{N}(\cdot)$, i.e., $\mathcal{S}_n(\cdot)$, is defined as the amount of energy consumed by the PHEV n, i.e., $d_n(\cdot)$, and the amount of required energy, i.e., $\mathcal{E}_n^{max} - \mathcal{E}_n^{res}(\cdot)$. Mathematically,*

$$\mathcal{S}_n(\cdot) = \frac{d_n(\cdot)}{\mathcal{E}_n^{max} - \mathcal{E}_n^{res}(\cdot)}, \quad \forall n \in \mathcal{N}(\cdot) \tag{13}$$

where $\mathcal{E}_n^{max}$ is the maximum battery capacity of each PHEV n, and $\mathcal{E}_n^{res}(\cdot)$ is the amount of stored energy present in the battery of PHEV n.

Each PHEV $n \in \mathcal{N}(\cdot)$ requests the energy-cloud service provider to supply $d_n(\cdot)$ amount of energy to maximize its satisfaction factor. If $PHEV_1$ and $PHEV_2$ consume $d_1(\cdot)$ and $d_2(\cdot)$ amount of energy, respectively, while their energy requirements are same, the PHEV consumes higher amount of energy, has higher satisfaction level. Mathematically,

$$\mathcal{S}_1(\cdot) \geq \mathcal{S}_2(\cdot), \quad \text{if } d_1 \geq d_2, \text{ and } [\mathcal{E}_1^{max} - \mathcal{E}_1^{res}(\cdot)] = [\mathcal{E}_2^{max} - \mathcal{E}_2^{res}(\cdot)] \tag{14}$$

Therefore, the utility function of PHEV $n \in \mathcal{N}(\cdot)$, i.e., $\phi_n(\cdot)$, must satisfy the inequalities as discussed below:

(i) The utility function of each PHEV $n \in \mathcal{N}(\cdot)$, $\phi_n(\cdot)$, is considered to be a non-decreasing function, as each PHEV n tries to consume high amount of energy, $d_n(\cdot)$, to maximize its satisfaction level, $\mathcal{S}_n(\cdot)$. We consider that the amount of energy requested to the energy-cloud service provider changes from $d_n(\cdot)$ to $\hat{d}_n(\cdot)$. Here, $d_n(\cdot)$ and $\hat{d}_n(\cdot)$ represent the current and new amount of requested energy by PHEV n. Hence,

$$\frac{\delta \phi_n(\cdot)}{\delta \hat{d}_n(\cdot)} \geq 0 \tag{15}$$

(ii) At marginal condition, the utility function of each PHEV n, $\phi_n(\cdot)$, is considered to be decreasing. Therefore, each PHEV n does not increase the amount of requested energy, $\hat{d}_n(\cdot)$, on reaching the marginal condition. Mathematically,

$$\frac{\delta^2 \phi_n(\cdot)}{\delta[\hat{d}_n(\cdot)]^2} < 0 \tag{16}$$

(iii) The amount of requested energy, $\hat{d}_n(\cdot)$, decreases with the increase in the price per unit energy, $p_n(\cdot)$. Therefore, with the increase in price per unit energy, $p_n(\cdot)$, the utility of each PHEV n, $\phi_n(\cdot)$ decreases. Mathematically,

$$\frac{\delta \phi_n(\cdot)}{\delta p_n(\cdot)} < 0 \tag{17}$$

We consider that the revenue function of each PHEV n, $\mathscr{R}_n(\cdot)$, is a concave function. Hence, we define the revenue function, $\mathscr{R}_n(\cdot)$, of PHEV n as follows:

$$\mathscr{R}_n(\cdot) = \mathcal{E}_n^{max} \tan^{-1}\left(e^{-\frac{\hat{d}_n(\cdot)-d_n(\cdot)}{d_n(\cdot)}}\right) \tag{18}$$

The cost function of PHEV n, $\mathscr{C}_n(\cdot)$, is defined as a linear function of amount of requested energy, $\hat{d}_n(\cdot)$, with price coefficient $p_n(\cdot)$, i.e., $\mathcal{P}(\cdot)$, defined in Equation (10). Mathematically,

$$\mathscr{C}_n(\cdot) = p_n(\cdot)\hat{d}_n(\cdot) \tag{19}$$

Therefore, considering the Equation (12), we define the utility function, $\phi_n(\cdot)$, of each PHEV n as follows:

$$\phi_n\left(\hat{d}_n(\cdot), \boldsymbol{d}_{-n}(\cdot), p_n(\cdot)\right) = \mathcal{E}_n^{max} \tan^{-1}\left(e^{-\frac{\hat{d}_n(\cdot)-d_n(\cdot)}{d_n(\cdot)}}\right) - p_n(\cdot)\hat{d}_n(\cdot) \tag{20}$$

where $\boldsymbol{d}_{-n}(\cdot) = \{d_1(\cdot), \cdots, d_{n-1}(\cdot), d_{n+1}, \cdots, d_{|\mathcal{N}(\cdot)|}(\cdot)\}$.

Lemma 1. *The satisfaction level of each PHEV n, i.e., $\mathcal{S}_n(\cdot)$, holds the following constraint:*

$$0 < \mathcal{S}_n(\cdot) \leq 1 \tag{21}$$

Proof: As we assume that each PHEV n requests an amount of energy, $\hat{d}_n(\cdot)$, that is positive. Mathematically,

$$d_n(\cdot) > 0 \tag{22}$$

Hence, the satisfaction level of each PHEV $n \in \mathcal{N}(\cdot)$, i.e., $\mathcal{S}_n(\cdot)$, follows the following inequality:

$$\begin{aligned} \mathcal{S}_n(\cdot) &= \frac{\hat{d}_n(\cdot)}{\mathcal{E}_n^{max} - \mathcal{E}_n^{res}} \\ \mathcal{S}_n(\cdot) &> 0, \quad \text{as } \hat{d}_n(\cdot) > 0 \end{aligned} \tag{23}$$

Each PHEV does not consume excess energy than its maximum battery capacity, as that results in increase the temperature of the battery, and shorten the lifetime of the battery. Hence, the amount of requested energy, $\hat{d}_n(\cdot)$, must satisfy the following inequality:

$$\hat{d}_n(\cdot) \leq \mathcal{E}_n^{req}(\cdot) = \mathcal{E}_n^{max} - \mathcal{E}_n^{res}(\cdot) \tag{24}$$

where $\mathcal{E}_n^{req}(\cdot)$ is the maximum amount of required energy to charge-fully the battery of PHEV n. Therefore,

$$\mathcal{S}_n(\cdot) \leq 1 \tag{25}$$

Therefore, the satisfaction level of each PHEV $n \in \mathcal{N}(\cdot)$, i.e., $\mathcal{S}_n(\cdot)$, satisfies the condition: $0 < \mathcal{S}_n(\cdot) \leq 1$ ∎

b) Utility function of energy-cloud service provider: The utility function of energy-cloud service provider, i.e., $\varphi(\cdot)$, signifies the earned capital of the energy-cloud service provider by supplying $\hat{d}_n$ amount of requested energy to the PHEV $n \in \mathcal{N}(\cdot)$. By supplying $\hat{d}_n(\cdot)$ amount of energy to each PHEV n with price per unit energy, $p_n(\cdot)$, the energy-cloud service provider earns $\hat{d}_n(\cdot)p_n(\cdot)$ amount of capital. Therefore, the total amount of earned capital of energy-cloud service provider is defined as follows:

$$\varphi(\cdot) = \sum_{n=1}^{n\in\mathcal{N}(\cdot)} \hat{d}_n(\cdot)p_n(\cdot) \tag{26}$$

Considering Equation (11), we rewrite Equation (26) as follows:

$$\varphi\left(\mathcal{P}(\cdot), \hat{d}_n(\cdot)\right) = \mathcal{P}(\cdot) \sum_{n=1}^{n\in\mathcal{N}(\cdot)} \hat{d}_n(\cdot) \tag{27}$$

The energy-cloud service provider tries to maximize its revenue by increasing the payoff of the utility function $\varphi(\cdot)$. Hence, the main objective of the energy-cloud service provider is as follows:

$$\arg\max \varphi\left(\mathcal{P}(\cdot), \hat{d}_n(\cdot)\right) \tag{28}$$

B. Existence of Generalized Nash Equilibrium

We determine the generalized Nash equilibrium for virtual energy-cloud topology control game in the proposed scheme, VELD, using the variational inequality condition, as discussed in Theorem 1.

Theorem 1. *Given the pricing function of the energy-cloud service provider, i.e., $\mathcal{P}(\cdot)$, there exists a variational equilibrium, i.e., generalized Nash equilibrium, for the utility function, $\phi_n(\cdot)$, for each PHEV $n \in \mathcal{N}(\cdot)$, and the condition for generalized nash equilibrium is as follows:*

$$\phi_n\left(\hat{d}_n^*(\cdot), \boldsymbol{d}_{-n}^*(\cdot), p_n(\cdot)\right) \geq \phi_n\left(d_n(\cdot), \boldsymbol{d}_{-n}^*(\cdot), p_n(\cdot)\right) \tag{29}$$

Proof: We know that the utility function of each PHEV n, i.e., $\phi_n(\cdot)$, needs to be maximized in order to reach the generalized Nash equilibrium. Hence, applying Karush-Kuhn-Tucker condition, we try to find out the variational equilibrium solution. Hence, we get:

$$\nabla_n \phi_n(\cdot) = 0 \tag{30}$$

Therefore, considering the overall utility function of the macro-grid, we can rewrite Equation (30) as follows:

$$\nabla \sum_{n\in\mathcal{N}(\cdot)} \phi_n(\cdot) = 0 \tag{31}$$

By performing the Jacobian transformation of the matrix derived by first-order derivative on Equation (31), we get a non-positive diagonal matrix. Hence, there exists a variational equilibrium for the proposed scheme, VELD. Therefore, we conclude that the proposed scheme, VELD, holds a generalized Nash equilibrium solution. ∎

C. The Proposed Algorithms

For virtual energy-cloud topology control using the proposed scheme, VELD, we propose two different algorithms

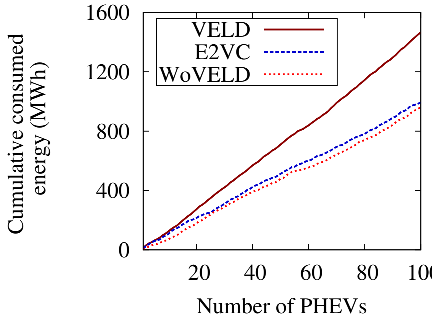

Fig. 4: Energy consumed by the PHEVs

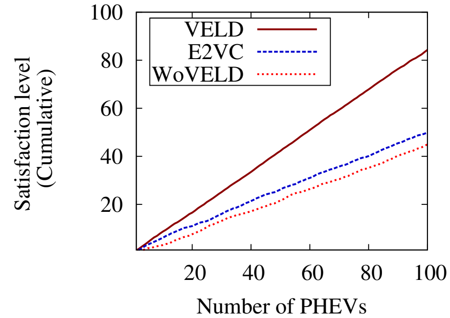

Fig. 5: Satisfaction level of the PHEVs

— for each PHEV, and for the energy-cloud service provider, as discussed in Algorithms 1 and 2, respectively. Using Algorithm 1, each PHEV $n \in \mathcal{N}(\cdot)$ decides the optimum amount of energy to be requested to the energy-cloud service provider. Based on the requested energy by the $\mathcal{N}(\cdot)$ PHEVs, the energy-cloud service provider decides the price per unit energy using Equation (10).

Algorithm 1 VELD algorithm for each PHEV

Inputs:
E_n^{max}: Maximum battery capacity of PHEV $n \in \mathcal{N}(\cdots)$
$d_n(\cdot)$: Previous amount of energy requested by PHEV n
$\mathcal{P}(\cdot)$: Price decided by the energy-cloud service provider
Output:
$\hat{d}_n(\cdot)$: Current amount of energy requested by PHEV n
Steps:
1. Decide the current amount of energy to be requested using following equation:
2. $\hat{d}_n(\cdot) = d_n(\cdot) + 0.01$ // `Energy request incremented by 0.01` kWh
if $(\phi_n \left(\hat{d}_n^*(\cdot), \boldsymbol{d}_{-n}^*(\cdot), p_n(\cdot)\right) \geq \phi_n \left(d_n(\cdot), \boldsymbol{d}_{-n}^*(\cdot), p_n(\cdot)\right))$
2.1. Request $\hat{d}_n(\cdot)$ amount of energy to energy-cloud service provider
else
2.2. Request $d_n(\cdot)$ amount of energy to energy-cloud service provider
// `Nash equilibrium reached`
end if

V. Performance Evaluation

A. Simulation Parameters

We consider that the PHEVs follow the Gauss-Markov mobility model, and moves in a two-dimensional plane simulated in MATLAB-based simulation platform. The PHEVs request the energy-cloud service provider to supply energy based on their requirements, considered as random value as shown in Table I.

B. Benchmark

The performance of the proposed scheme, virtual energy cloud topology control (VELD), is evaluated by comparing the results with other energy distribution policies such as the economics of electric vehicle charging (E2VC) [4], and the energy distribution without any game-theoretic approach

Algorithm 2 VELD algorithm for each energy-cloud service provider

Inputs:
$\hat{d}_n(\cdot)$: Current amount of energy requested by each PHEV $n \in \mathcal{N}(\cdot)$
$c_m(\cdot)$: Energy generation cost per micro-grid $m \in \mathcal{M}$
Output:
$\mathcal{P}(\cdot)$: Price decided by the energy-cloud service provider
Steps:
1. Calculate $\mathcal{D}(\cdot) = \sum_n \hat{d}_n(\cdot)$
2. Calculate average energy generation cost, c^{avg}, using following equation:
$c^{avg} = \frac{\sum_m c_m(\cdot)}{|\mathcal{M}|}$
3. Decide the new price unit energy, $\mathcal{P}(t)$, using the following equation:
$\mathcal{P}(t) = c^{avg} + \tan^{-1}(e^{\mathcal{D}(\cdot)})$
4. Broadcast the new price per unit energy, $\mathcal{P}(t)$

TABLE I: Simulation Parameters

Parameter	Value
Simulation area	10 $km \times$10 km
Number of PHEVs	100
Maximum battery capacity	35-65 MWh
Residual energy of each PHEV	>10 MWh
Excess energy per micro-grid	99 MWh

(WoVELD). We refer to these different energy trading policies as VELD, E2VC, and WoVELD, through the rest of the paper. In E2VC [4], the authors proposed a non-cooperative game theoretic approach. Though the authors did not consider the choice of any energy-cloud service provider for the PHEVs available in the coalition. In WoVELD, we considered that each PHEV requests the energy-cloud service provider based on their requirements. Thus, we can improve the satisfaction factor of the PHEVs, and the energy load to the micro-grids using our proposed scheme, VELD, than using other approaches, i.e., E2VC and WoVELD.

C. Results and Discussions

For simulation, we assume that the energy-cloud service provider calcualtes the real-time supply and demands in every 5 seconds interval. Figure 4 shows that the amount of energy consumed by the PHEVs is higher using the proposed

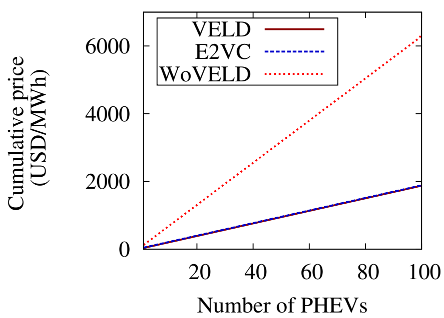

Fig. 6: Price for the PHEVs

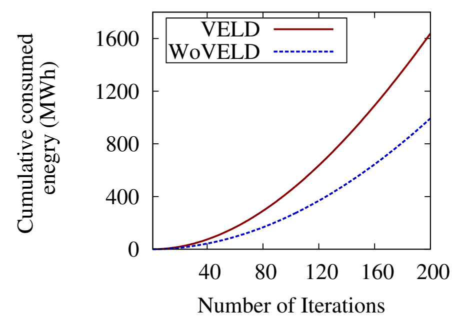

Fig. 7: Energy consumed per iteration

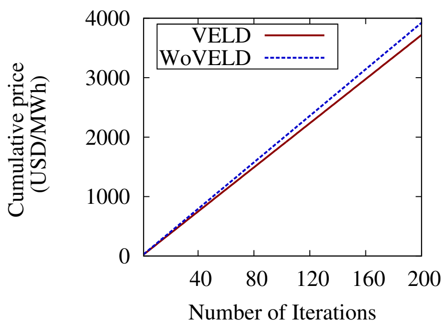

Fig. 8: Price per iteration

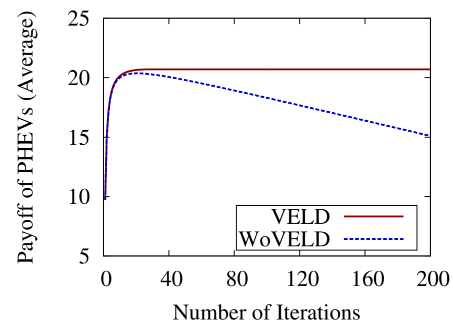

Fig. 9: Utility per iteration

scheme, VELD, than using E2VC and WoVELD. Therefore, the satisfaction levels of the PHEVs are much higher using the proposed scheme, VELD, than using the other schemes such as E2VC and WoVELD, as shown in Figure 5. In Figure 6, we evaluated the cumulative price per unit energy which is almost same using the proposed scheme, VELD, and E2VC, and higher using WoVELD. Using the proposed scheme, VELD, the PHEVs consume 47.49% and 52.96% higher amount of energy than using E2VC and WoVELD, respectively.

Figure 7 shows an incremented curve with cumulative energy consumed per iteration. From Figure 7, we conclude that the energy consumed per iteration is 64.87% higher using VELD than using WoVELD. The average price per unit energy in each iteration is also 5.52% lower using VELD than using WoVELD, as shown in Figure 8. Using the propose scheme, VELD, the payoff of the utility function is always equal or higher than using WoVELD, as shown in Figure 9.

VI. Conclusion

In this paper, we formulated a single leader multiple follower Stackelberg game based VELD scheme to study the problem of energy distribution using virtual energy cloud. Based on the proposed scheme, VELD, we showed how each PHEV consumes high amount of energy with paying less price per amount of energy. The simulation results show that the proposed scheme, VELD, yields improved results.

Future extension of this work includes understanding of how the energy redistribution can be done in virtual energy cloud infrastructure by the energy-cloud service provider.

References

[1] S. Bera, S. Misra, and J. Rodrigues, "Cloud Computing Applications for Smart Grid: A Survey," *IEEE Transactions on Parallel and Distributed Systems*, 2014, doi: 10.1109/TPDS.2014.2321378.

[2] A. Mondal and S. Misra, "Dynamic Coalition Formation in a Smart Grid: A Game Theoretic Approach," in *Proceedings of IEEE International Workshop on Smart Communication Protocols and Algorithms in conjunction with IEEE International Conference on Communications*, Budapest, Hungary, June 2013.

[3] S. Misra, S. Bera, and T. Ojha, "D2P: Distributed Dynamic Pricing Policy in Smart Grid for PHEVs Management," *IEEE Transactions on Parallel and Distributed Systems*, 2014, doi: 10.1109/TPDS.2014.2315195.

[4] W. Tushar, W. Saad, H. Poor, and D. Smith, "Economics of Electric Vehicle Charging: A Game Theoretic Approach," *IEEE Transactions on Smart Grid*, vol. 3, no. 4, pp. 1767–1778, December 2012.

[5] B. Brandherm, J. Baus, and J. Frey, "Peer Energy Cloud - Civil Marketplace for Trading Renewable Energies," in *the* 8^{th} *International Conference on Intelligent Environments*, June 2012, pp. 375–378.

[6] S. Misra, P. Venkata Krishna, V. Saritha, H. Agarwal, and A. Ahuja, "Learning Automata-based Multi-constrained Fault-tolerance Approach for Effective Energy Management in Smart Grid Communication Network," *Journal of Network and Computer Applications*, vol. 44, pp. 212–219, September 2014.

[7] S. Misra, S. Bera, and M. S. Obaidat, "Economics of Customer's Decisions in Smart Grid," *IET Networks*, Apr. 2014, doi: 10.1049/iet-net.2013.0182, 2014.

[8] S. Misra, P. V. Krishna, V. Saritha, and M. S. Obaidat, "Learning Automata as a Utility for Power Management in Smart Grids," *IEEE Communications Magazine*, vol. 51, no. 1, pp. 98–104, 2013.

[9] N. Kumar, S. Misra, and M. Obaidat, "Routing as a Bayesian Coalition Game in Smart Grid Neighborhood Area Networks: Learning Automata-based Approach," in *IEEE International Conference on Communications*, June 2014, pp. 1502–1507.

[10] H. Kim, Y.-J. Kim, K. Yang, and M. Thottan, "Cloud-based Demand Response for Smart Grid: Architecture and Distributed Algorithms,"

in *IEEE International Conference on Smart Grid Communications*, October 2011, pp. 398–403.

[11] S. Misra, A. Mondal, S. Banik, M. Khatua, S. Bera, and M. Obaidat, "Residential Energy Management in Smart Grid: A Markov Decision Process-Based Approach," in *IEEE International Conference on Internet of Things*, August 2013, pp. 1152–1157.

[12] X. Li and J.-C. Lo, "Pricing and Peak Aware Scheduling Algorithm for Cloud Computing," in *IEEE PES Innovative Smart Grid Technologies*, January 2012, pp. 1–7.

[13] S. Misra, S. Das, M. Khatua, and M. Obaidat, "QoS-Guaranteed Bandwidth Shifting and Redistribution in Mobile Cloud Environment," *IEEE Transactions on Cloud Computing*, vol. 2, no. 2, pp. 181–193, April 2014.

[14] F. Luo, Z. Y. Dong, Y. Chen, Y. Xu, K. Meng, and K. P. Wong, "Hybrid Cloud Computing Platform: The Next Generation IT Backbone for Smart Grid," in *IEEE Power and Energy Society General Meeting*, July 2012, pp. 1–7.

[15] T. Rajeev and S. Ashok, "A Cloud Computing Approach for Power Management of Microgrids," in *IEEE PES Innovative Smart Grid Technologies*, December 2011, pp. 49–52.

[16] M. Such and C. Hill, "Battery Energy Storage and Wind Energy Integrated into the Smart Grid," in *Proceedings of IEEE PES on Innovative Smart Grid Technologies*, Washington, DC, January 2012, pp. 1–4.

[17] B. Liang and Z. Haas, "Predictive Distance-based Mobility Management for PCS Networks," in *Proceedings of IEEE INFOCOM*, vol. 3, March 1999, pp. 1377–1384.