DEMANDS: Distributed Energy Management Using Non-cooperative Scheduling in Smart Grid

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***Abstract*—In this paper, the problem of energy scheduling and energy exchange between micro-grids and customers is studied as a multi-leader multi-follower non-cooperative Stackelberg game. The customers act as leaders, and decide the amount of required energy to be taken in each time slot. On the other hand, the micro-grids act as the followers, which need to decide the price per unit energy based on the total requested energy by the connected customers. Using variational inequality, it is shown that the proposed *distributed energy management using scheduling* (DEMANDS) scheme has a Nash equilibrium solution, which is also socially optimal. In the proposed scheme, DEMANDS, each customer gets energy from any of the available micro-grids within a coalition neither by paying higher price per unit energy, nor by waiting for the next time slot for service. The proposed scheme, DEMANDS, which enables the micro-grids and the customers to reach the equilibrium state, is evaluated theoretically as well as through simulations.**

***Index Terms*—Distributed energy management, Scheduling, Smart grid, Micro-grid, Stackelberg game, non-cooperative game.**

I. Introduction

To achieve improved quality of service, traditional electrical grids are being modernized as *smart grids*. A smart grid [1] is conceptualized as a cyber-physical system equipped with different sustainable models — energy production, energy distribution, and energy usage. A smart grid also integrates several advanced techniques such as advanced metering infrastructure, energy management system, distributed energy system, intelligent electronic devices, and plug-in hybrid electrical vehicles [1]. Unlike the traditional grid, in which electricity is distributed unidirectionally to the customers having a centralized system, in smart grid, customers can also participate in the distribution of energy by announcing the actual amount of required energy using duplex communication infrastructure. In smart grid, the large-scale electrical grid is divided into smaller geographical areas [2], [3]. The energy demand of each geographical area is fulfilled by single or multiple micro-grids [4] having bi-directional electricity exchange facilities with the substation, and the main grid. In the presence of several micro-grids [4], it is desired to allow the customer to choose appropriate micro-grids to ensure proper distribution of energy with lower price. If insufficient amount of energy is generated by the micro-grids, the customers try to optimize their requested amount of energy, while utilizing the stored energy. Additionally, customers seek opportunity to consume energy from the micro-grids, if excess energy is generated by the micro-grids, in a time slot. Therefore, a distributed energy management system is required to ensure quality of energy service for each micro-grid and the overall smart grid infrastructure. One of the important features in a smart grid is the demand-side energy distribution, which gives the opportunity for flexible energy demand according to the requirements of the customers, and the micro-grid decides the price per unit energy based on the energy demand.

The micro-grids generate energy typically using renewable energy resources [5] such as wind power, solar energy, and hydro power. So, the amount of generated energy is not fixed at different times in a day. If a micro-grid has excess amount of energy, it sells that excess amount to the main grid, or other micro-grids having demands of energy. As the requested energy by the customers to each micro-grid is discrete, the load on each micro-grid does not remain the same in any specific time. Moreover, the existing literature on energy distribution in smart grid considered different energy management schemes, using which each customer is connected with a single micro-grid. Thus, in on-peak hours, if the customer is willing to pay a higher price, s/he gets the requested energy at that time slot; otherwise, s/he waits for a significant duration of time to get serviced. To overcome this problem, we need proper *distributed energy management using a scheduling* approach. Consequently, the customers need to decide the amount of energy to be requested to the micro-grid in each time slot. On the other hand, the micro-grids need to decide the price per unit energy to ensure maximum profit. In order to do that, the customers and the micro-grids need to maximize the respective payoff values of the utility functions.

In this paper, we propose a non-cooperative game theoretic algorithm, named DEMANDS, for *distributed energy management using scheduling*. We use a multi-leader multi-follower Stackelberg game theory to decide the strategies of the customers to fulfill their energy demand while expending lower cost. On the other hand, the micro-grids choose strategies to maximize their profit and properly utilize the generated energy. We summarize the contributions of this paper as follows:

a) We propose an algorithm for *distributed energy management using scheduling* for real-time energy consumption in the presence of multiple micro-grids in a coalition. Each customer or micro-grid, decides his/her/its strategy, based on the local information. Thus, the proposed algorithm, DEMANDS, is distributed, which makes it less vulnerable to system failures.

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b) The multi-leader multi-follower Stackelberg game is used to evaluate the optimal strategies of the customers using a non-cooperative game, and, in the next stage, the optimal strategies of the micro-grids are also decided using another similar approach.

c) We present two different algorithms. The first one is executed at the customer-end to determine the amount of energy to be requested. Each micro-grid performs the second algorithm in a non-cooperative manner, and decides the price per unit energy in a distributed way, based on the total amount of requested energy.

II. Related Work

In the last few years, lot of research work on smart grid emerged [6]–[21]. Some of the existing literature are discussed in this Section. Mondal and Misra [9] proposed a decision making process to form coalitions dynamically between micro-grids and customers. However, they did not consider how energy requirement can be scheduled in a distributed manner from the customer-side. Bakker *et al.* [10] recommended a distributed load management scheme with dynamic pricing, and have modeled it as a network congestion game. However, in this scenario, customer has to wait, as the customer does not have the option to switch to another micro-grid. Molderink *et al.* [11] proposed an algorithm by using the energy in the off-peak, and the on-peak hours, with a virtual power plant, for energy management. Sanseverino *et al.* [12] studied an algorithm for load shifting and storage device management. The authors proposed that during peak-hour, heavy loads should be turned off, and vice-versa. However, in this scenario, if the customer does not want any delay in receiving the requested energy, s/he has to pay high price for it. In this situation, using the scheme proposed in the present paper, i.e., DEMANDS, the customer can get energy from the other micro-grids with lower price per unit energy, and with much lower delay.

Vytelingum *et al.* [13] proposed an algorithm, in which the customers choose their strategies based on their advance knowledge about the market. They did not consider scheduling in distributed energy management. Fang *et al.* [22] proposed different energy management schemes. However, in this work, new opportunities for improved residential energy management and bill reduction are studied without considering the impact of scheduling approach for distributed energy management. Erol-Kantarci and Mouftah [23] studied a time-to-use (TOU) aware-energy management scheme. In this scheme, a customer consumes energy according to the time. Yet, the energy management policy adopted by the customers and the micro-grids need further research to have an optimal solution and with minimum delay and less message overhead.

Pipattanasomporn *et al.* [24] considered smart grid as a multi-agent system, which is visualized to be a combination of several agents working in cooperation to achieve a goal. The authors decompose the complex problem into multiple fragments, namely, control, distributed energy resource (DER), user and database. However, in this paper, we consider an oligopolistic market, where multiple customers and multiple micro-grids act non-cooperatively, while attempting to maximize his/her/its own payoff, individually.

In contrast to the existing works, a model is used in this paper to characterize the effect of distributed energy management using scheduling in the smart grid. We use the multi-leader multi-follower Stackelberg game to develop an optimal solution for distributed energy management using scheduling for the customers, where each customer has multiple option for choosing micro-grids.

III. System Model

We consider a distributed energy management system with multiple micro-grids and multiple customers. Each customer connected with multiple micro-grids is visualized using relays in the electrical network. Each customer has a dedicated relay to switch connection between the available micro-grids which are available with the coalition. We consider that each customer is not connected with multiple micro-grids or main grid, simultaneously. The schematic diagram of the proposed scheme, DEMANDS, is shown in Figure 1. At each time slot, each customer chooses one suitable micro-grid and consumes energy from the selected micro-grid among the available micro-grids in the coalition. On the other hand, each micro-grid does not depend on other micro-grids for energy transmission to the customers-end. We consider that, in this system, each micro-grid $m \in \mathbb{M}$, where $\mathbb{M}$ is the set of micro-grids available in a coalition $\mathbb{C}$, serves the electricity demand. Each customer $n \in \mathbb{N}$, where $\mathbb{N}$ is the set of the customers in coalition $\mathbb{C}$, demands x_n^t amount of energy to the micro-grid m in time slot $t \in \mathbb{T}$, where $\mathbb{T}$ is the set of the time slots in a day. Therefore, in time slot t, the total energy demanded from the micro-grid m by the customers $\mathbb{N}_m \subseteq \mathbb{N}$, where $\mathbb{N}_m$ is the set of the customers who request energy to the micro-grid m, is $\mathbb{D}_m^t$. Mathematically,

$$\mathbb{D}_m^t = \sum_{n \in \mathbb{N}_m}^{\mathbb{N}_m \subseteq \mathbb{N}} x_n^t, \quad \forall t \in \mathbb{T} \tag{1}$$

The total demand to micro-grid m by the customers $\mathbb{N}_m$ must satisfy the following inequality,

$$\mathbb{D}_m^t \leq \mathbb{G}_m^t, \quad \forall m \in \mathbb{M} \tag{2}$$

where $\mathbb{G}_m^t$ is the total generation capacity of the micro-grid m in the time slot t.

Given the amount of energy requested by the customers $\mathbb{N}_m$, each micro-grid m sets a price p_m^t to maximize its revenue from supplying energy by strategically choosing the optimal value for its price coefficient.

For completing energy trading successfully, the customers and the micro-grids exchange messages with one another, and agree on the energy trading parameters — the amount of required energy by each customer n, x_n^t, and the price per unit energy for micro-grid m, p_m^t, that satisfy the objectives of both players, i.e., the customers and the micro-grids. We consider that in a day, the amount of required energy by each customer n, e_n, is predicted on a day-ahead basis, based on the prediction of the maximum energy requirement by the appliances installed on the customer-side [25]. However, x_n^t, is determined by the trade-off between the total required energy of each customer n, and p_m^t decided by the micro-grid m. We represent the

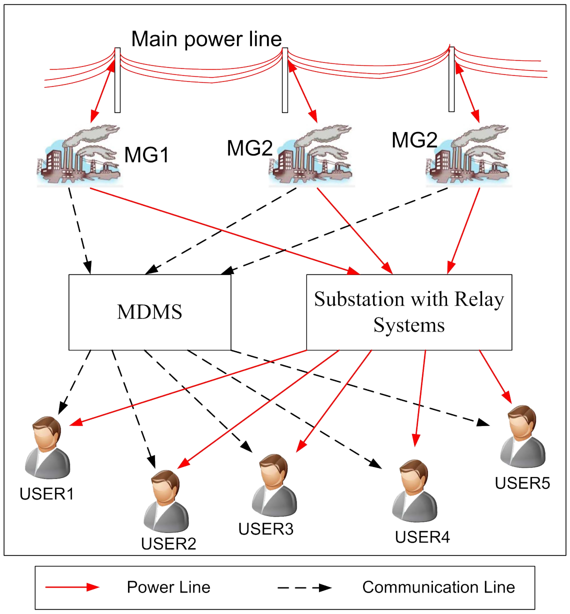

Fig. 1: Schematic Diagram of DEMANDS

price vector, $\vec{\mathbb{P}}^t$, as the collection of price per unit energy, p_m^t, as defined in Definition 1. The demanded energy by each customer n in time slot t, i.e., x_n^t, must satisfy the following constraints:

$$e_n \leq \sum_{t=1}^{t\in\mathbb{T}} x_n^t, \text{ and } \sum_{m=1}^{m\in\mathbb{M}} \sum_{t=1}^{t\in\mathbb{T}} \mathbb{G}_m^t + \Delta_m \geq \sum_{n=1}^{n\in\mathbb{N}} \sum_{t=1}^{t\in\mathbb{T}} x_m^t \quad (3)$$

where Δ_m is the amount of energy consumed by the micro-grid m from the main grid. Moreover, the price per unit energy decided by the micro-grid m, p_m^t, is dependent on the total amount of demanded energy by the customer $\mathbb{N}_m$, i.e., $\sum_{n\in\mathbb{N}_m} x_n^t$, and the number of customers requested energy at that time slot t, i.e., $|\mathbb{N}_m^t|$. The energy generation by each micro-grid depends on the renewable energy resources and external environmental factors. Hence, the amount of generated energy cannot be modified for a fixed time slot. Thereby, the total generation capacity of the micro-grids $\mathbb{M}$ within a coalition, i.e., $\sum_{m=1}^{m\in\mathbb{M}} \mathbb{G}_m^t$, is unchangeable for a time slot t, as well as, the individual generation capacity of each micro-grid m for a time slot t, $\mathbb{G}_m^t$, is also unchangeable. However, $\sum_{m=1}^{m\in\mathbb{M}} \mathbb{G}_m^t$ and $\mathbb{G}_m^t$ are not constants. Therefore, if a micro-grid m needs excess amount of energy, it needs to request the main grid for deficient amount of energy.

Definition 1. *For each time slot t, the price vector $\vec{\mathbb{P}}^t$ is defined by the vector with components having information about the price per unit energy decided by each micro-grid m. Mathematically, we define the price vector $\vec{\mathbb{P}}^t$ as* — $\vec{\mathbb{P}}^t = \{p_1^t, p_2^t, \cdots, p_m^t, \cdots, p_{|\mathbb{M}|}^t\}^T, \quad \forall t \in \mathbb{T}$

Therefore, the energy requested by each customer n has to fulfill the inequalities given in Equations (2)-(3). It also affects the price per unit energy decided by each micro-grid m. Thus, the main challenges facing the development of *distributed energy management using scheduling* (DEMANDS) approach are:

i) Modeling the decision making processes, and the interaction between the micro-grids and the customers.

ii) Developing an algorithm, i.e., DEMANDS, for customers such that they can decide the optimum energy to be requested in each time slot t, given the price vector $\vec{\mathbb{P}}^t$ decided by the micro-grids $\mathbb{M}$.

iii) Each micro-grid m decides the price per unit energy, i.e., p_m^t, based on the total demanded energy, $\mathbb{D}_m^t$, while ensuring revenue maximization.

Communication between Micro-grids and Customers: We assume that the communication infrastructure between the customers and the micro-grids is based on wireless mesh network (WMN). We use the IEEE 802.11b protocol for communication between the micro-grids and the customers. Firstly, each customer decides the amount of energy to be requested, and sends a request message. The request message format is shown in Figure 2. Based on the requests by the customers, each micro-grid decides the price per unit energy, and sends a reply message back to the customers. The reply message format is shown in Figure 3.

Type (Request)	CustomerID	Energy (MWh)	SelectionFlag
1 Byte	2 Byte	1 Byte	2 Byte

Fig. 2: Request message

Type (Reply)	GridID	Price (USD)	Generated Energy (MWh)
1 Byte	2 Byte	2 Byte	2 Byte

Fig. 3: Reply message

IV. DEMANDS: The Proposed Noncooperative Game Theoretic Solution

A. Justification for the Use of Multi-Leader Multi-Follower Stackelberg Game

In a distributed energy management scenario, each customer tries to consume high amount of energy in order to fulfill his/her energy requirement. However, having option for connecting one of the multiple micro-grids available in the coalition, s/he needs to decide the amount of energy to be consumed, while paying less. On the other hand, energy requested by each customer has a cumulative effect on the price decided by the micro-grids. Thereby, we use a *multi-leader multi-follower Stackelberg game*, where the customer act as the leaders, and the micro-grids act as the followers. This interaction has a similarity with 'oligopolistic market', where the request and supply trade-off is to be maintained in order to maximize individual profit of each player.

B. Game formulation

To study the interaction between the micro-grids and the customers, we use a multi-leader multi-follower Stackelberg game [26]. Multi-leader multi-follower Stackelberg game is an multi-stage and multi-level game. Here, the customers, who act as leaders, decide the energy to be consumed from which micro-grid based on the price vector defined by the micro-grids, independently. On the other hand, the micro-grids act as the followers, and decide the price per unit energy based on the total demanded energy, independently. In this paper, we follow an extended game formulation approach used by Tushar *et al.* [27]. We define the strategic form, $\boldsymbol{\xi}$, of the proposed non-cooperative game as follows:

$$\boldsymbol{\xi} = \langle (\mathbb{N} \cup \mathbb{M}), (e_n, x_n^t, \mathscr{U}_n)_{n\in\mathbb{N}}, (p_m^t, \mathscr{B}_m)_{m\in\mathbb{M}}, (\vec{\mathbb{P}}^t)_{t\in\mathbb{T}} \rangle \quad (4)$$

The components of strategic form $\boldsymbol{\xi}$ are as follows:

i) Each customer n acts as the leader, and decides x_n^t while satisfying the constraint defined in Equation (3).
ii) The utility function $\mathscr{U}_n$ of each customer n captures the benefit of consuming x_n^t amount of energy in each time slot t.
iii) The utility function $\mathscr{B}_m$ of each micro-grid m captures the revenue gained by supplying the total demanded energy, $\mathbb{D}_m^t$, by the $\mathbb{N}_m$ set of customers.
iv) The price per unit energy p_m^t is decided by micro-grid m for each time slot t. It is also dependent on the total amount of demanded energy by the $\mathbb{N}_m$ set of customers.
v) Price vector $\vec{\mathbb{P}}^t$ is defined as a vector having the information about the price per unit energy decided by the $\mathbb{M}$ micro-grids for time slot t within a coalition.

1) Utility function of a Customer: For each customer n, we define a utility function, $\mathscr{U}_n(x_n^t, \mathbf{x}_{-n}^t, p_m^t)$, to represent the quantified benefit consuming x_n^t in each time slot t. Here, each customer n tries to maximize his/her individual energy consumption, while paying less money. Hence, each customer maximizes his/her payoff of the utility function $\mathscr{U}_n$. Thus, the properties that each customer n must satisfy, are as follows:

i) The utility function $\mathscr{U}_n$ of each customer n is considered to be a nondecreasing and nonnegative function while satisfying the constraint given in Equation (3).

ii) For the marginal value of energy consumption, $\mathscr{U}_n$ is considered to be a non-increasing function.

iii) p_m^t decided by micro-grid m affects the utility function of each customer n. With higher p_m^t, amount of demanded energy x_n^t decreases.

Therefore, $\mathscr{U}_n$ of each customer n is as follows:

$$\mathscr{U}_n(x_n^t, \mathbf{x}_{-n}^t, p_m^t) = e_n x_n^t - \frac{1}{2}\gamma_n^t (x_n^t)^2 - p_m^t x_n^t \tag{5}$$

where γ_n^t is the *satisfaction factor* of customer n at time slot t, as defined in Definition 2, and $\gamma_n^t \in (0,1]$, $x_n^t \in [0, \mathbb{G}_m^t - \sum_{i=1, i\neq n}^{i\in\mathbb{N}_m} x_i^t]$, $x_n^t \in [0, e_n - \sum_{\tau=1}^{t-1} x_n^\tau]$, and $\mathbf{x}_{-n}^t = \{x_1^t, x_2^t, \cdots, x_{n-1}^t, x_{n+1}^t, \cdots, x_{|\mathbb{N}_m|}^t\}$.

Definition 2. *At time slot t, the satisfaction factor of a customer n, γ_n^t, is a quantified value which is proportionate with the ratio of the total energy consumed till previous time slot $(t-1)$, i.e., $\sum_{\tau=1}^{t-1} x_n^\tau$, and the total required energy in $\mathbb{T}$ time slots, i.e., $\sum_{\tau=1}^{|\mathbb{T}|} x_n^\tau$ or e_n.*

From Definition 2, we conclude that satisfaction factor γ_n^τ of customer n at time slot τ is always the same or is higher than the satisfaction factor of customer n at time slot τ', $\gamma_n^{\tau'}$, where $\tau \geq \tau'$. Mathematically,

$$\gamma_n^\tau \geq \gamma_n^{\tau'}, \quad \text{if } \tau \geq \tau' \tag{6}$$

We assume that each customer n does not consume higher amount of energy than his/her requirements.

Lemma 1. *The satisfaction factor of each customer n can have the maximum value of 1. Mathematically,*

$$\arg\max_n \gamma_n^t \leq 1, \quad \forall t \in \mathbb{T} \tag{7}$$

Proof: As we assumed that the required energy of each customer n in a day, i.e., e_n, is fixed, and no customer demands higher amount of energy than his/her total requirement. Therefore,

$$\arg\max_{n\in\mathbb{N}} e_n \geq \arg\max_{n\in\mathbb{N}} \sum_{t=1}^{t\in\mathbb{T}} x_n^t \Rightarrow \arg\max_{n\in\mathbb{N}} [(\sum_{\tau=1}^{\tau\in\mathbb{T}} x_n^\tau)/(e_n)] \leq 1 \tag{8}$$

We know that $\gamma_n^t = \arg\max_{n\in\mathbb{N}} [(\sum_{\tau=1}^{\tau\in\mathbb{T}} x_n^\tau)/(e_n)]$. Hence, it is proved that — $\arg\max_n \gamma_n^t \leq 1, \quad \forall t \in \mathbb{T}$. ■

2) Utility function of a micro-grid: For each micro-grid m, we define a utility function $\mathscr{B}_m(p_m^t, x_n^t(p_m^t))$ to represent the quantified profit by selling $\sum_{n\in\mathbb{N}_m} x_n^t$ at time slot t. By supplying x_n^t, the micro-grid m makes a profit of $p_m^t x_n^t$ amount at time slot t. Each micro-grid m aims to maximize its revenue by selling the generated energy, $\mathbb{G}_m^t$. Thus, each micro-grid m satisfies following properties:

i) Each micro-grid m tries to increase the amount of selling energy, i.e., $\sum_{n\in\mathbb{N}_m} x_n^t$, as it increases the revenue, while satisfying the constraints given in Equations (1), and (2).

ii) For marginal revenue of a micro-grid m, $\mathscr{B}_m$ is considered to be a non-increasing function. For reaching this marginal revenue state, each micro-grid needs to satisfy the condition – $\mathbb{G}_m^t = \sum_{n\in\mathbb{N}_m} x_n^t, \quad \forall m \in \mathbb{M}$.

iii) Each micro-grid m tries to sell $\mathbb{G}_m^t$ with higher price to maximize the revenue.

Therefore, the utility function $\mathscr{B}_m$ for each micro-grid m is as follows:

$$\mathscr{B}_m(p_m^t, x_n^t(p_m^t)) = p_m^t \sum_{n\in\mathbb{N}_m} x_n^t \tag{9}$$

In the proposed DEMANDS algorithm, the customers control the price per unit energy indirectly, by choosing a micro-grid from the available micro-grids $\mathbb{M}$. However, the micro-grids decide the price per unit energy, p_m^t, where m, using a dynamic pricing model. We defined the dynamic pricing model as follows:

$$p_m^t = \begin{cases} c_m^t, & \text{if } (p_m^t < c_m^t) \text{ and } (\mathbb{G}_m^t \geq \sum_{n\in\mathbb{N}_m} x_n^t) \\ K, & \text{if } (p_m^t \geq c_m^t) \text{ and } (\mathbb{G}_m^t \geq \sum_{n\in\mathbb{N}_m} x_n^t) \\ \infty, & \text{if } (\mathbb{G}_m^t < \sum_{n\in\mathbb{N}_m} x_n^t) \end{cases} \tag{10}$$

where c_m^t is the generation cost per unit energy of micro-grid m at time slot t, and $K = \mathscr{A}_m(\sum_{n\in\mathbb{N}_m} x_n^t) + \mathscr{B}_m(\sum_{n\in\mathbb{N}_m} x_n^t)^2 + \mathscr{C}_m$, as defined in [28], where $\mathscr{A}_m$, $\mathscr{B}_m$, and $\mathscr{C}_m$ are constants for the micro-grid m. Here, each customer n tries to maximize his/her utility function $\mathscr{U}_n$, while satisfying the constraints in Equations (1), and (2). However, the energy demand of each customer n leads to the adoption of non-cooperative strategy. Therefore, to reach the generalized Nash equilibrium (GNE), each customer n chooses an amount of energy to be requested, such that,

$$\arg\max_{x_n^t} \mathscr{U}_n(x_n^t, \mathbf{x}_{-n}^t, p_m^t) \Rightarrow \arg\max_{x_n^t} [e_n x_n^t - \frac{1}{2}\gamma_n^t (x_n^t)^2 - p_m^t x_n^t] \tag{11}$$

However, each micro-grid m tries to choose an optimum price per unit energy. Therefore, to reach the GNE, each micro-grid m chooses a price per unit energy, p_m^t, such that,

$$\arg\max_{p_m^t} \mathscr{B}_m(p_m^t, x_n^t(p_m^t)) \Rightarrow \arg\max_{p_m^t}[\sum_{n\in\mathbb{N}_m} p_m^t x_n^t] \quad (12)$$

Hence, the solution of the proposed DEMANDS algorithm reaches the Stackelberg equilibrium at which all the leaders, i.e., $\mathbb{N}$ customers, reach their optimal amount of requested energy, given the followers' optimal strategies, i.e., the price per unit energy by $\mathbb{M}$ micro-grids at their GNE. We define the GNE states of the players, i.e., the followers and the leaders, as defined in Definition 3.

Definition 3. *We define the GNE of proposed strategic form $\boldsymbol{\xi}$ of DEMANDS algorithm using a non-cooperative game, if and only if, utility function of each customer n, i.e., leader, $\mathscr{U}_n(x_n^t, \mathbf{x}_{-n}^t, p_m^t)$, and the utility function of each micro-grid m, i.e., follower, $\mathscr{B}_m(p_m^t, x_n^t(p_m^t))$ satisfy following inequalities defined in Equation (13).*

$$\mathscr{U}_n(x_n^{t*}, \mathrm{x}_{-n}^{t*}, p_m^{t*}) \geq \mathscr{U}_n(x_n^t, \mathrm{x}_{-n}^{t*}, p_m^{t*}), \text{ and}$$
$$\mathscr{B}_m(p_m^{t*}, x_n^{t*}(p_m^{t*})) \geq \mathscr{B}_m(p_m^t, x_n^{t*}(p_m^t)) \quad (13)$$

where n, $\sum_{n\in\mathbb{N}_m} x_n^t \leq \mathbb{G}_m^t$, $x_n^{t*} \in \mathrm{x}^{t*}$, m, $p_m^{t*} \in \boldsymbol{p}^{t*}$, p_m^{t*} is the price per unit energy at Nash equilibrium decided by the micro-grid m for time slot t, and x_n^{t*} is the requested energy at Nash equilibrium decided by the customer n for time slot t.

C. Existence of generalized Nash equilibrium

In multi-player non-cooperative game, the existence of an equilibrium pure strategic solution is not guaranteed always. Hence, we need to determine the existence of GNE in our proposed multi-leader multi-follower game, i.e., DEMANDS. Due to the fact that variational equality is more socially stable than other GNE, as studied by Tushar *et al.* [27], we try to seek the variational equality for the customers, as discussed in Theorem 1.

Theorem 1. *If the set of price decided by micro-grid $\mathbb{M}$, i.e., $\vec{\mathbb{P}}^t$, is fixed for a time slot t, there exists a variational equality for utility function $\mathscr{U}_n(x_n^t, \mathrm{x}_{-n}^t, p_m^t)$.*

Proof: We define the overall utility function $\tilde{\mathscr{U}}$ of $\mathbb{N}_m$ customers as follows:

$$\tilde{\mathscr{U}}(\cdot) = \sum_{n=1}^{n\in\mathbb{N}_m}[e_n x_n^t - \frac{1}{2}\gamma_n^t(x_n^t)^2] - p_m^t \sum_{n=1}^{n\in\mathbb{N}_m} x_n^t. \quad (14)$$

Hence, to reach the GNE state, we need to maximize the overall utility function $\tilde{\mathscr{U}}$. Mathematically,

$$\arg\max \tilde{\mathscr{U}}(x_1^t, x_2^t, \cdots, x_n^t, \cdots, x_{|\mathbb{N}_m|}^t; p_m^t) \quad (15)$$

Applying the Karush-Kuhn-Tucker (KKT) conditions with Lagrange multipliers of each n^{th} customer, we get,

$$\nabla_{x_n^t}\mathscr{U}_n(x_n^t, \mathbf{x}_{-n}^t, p_m^t) - \lambda_n^t \nabla_{x_n^t}(\sum_{n\in\mathbb{N}_m} x_n^t - \mathbb{G}_m^t) = 0, \quad (16)$$

$$\lambda_n^t \nabla_{x_n^t}(\sum_{n\in\mathbb{N}_m} x_n^t - \mathbb{G}_m^t) = 0, \quad (17)$$

where $\lambda_n^t \geq 0$, and λ_n^t is the Lagrangian multiplier for the customer n at time slot t.

Hence, we extend Equation (16). Therefore, applying the KKT condition over the overall utility function $\tilde{\mathscr{U}}$, we get

$$\nabla_{\mathbf{x}^t}\tilde{\mathscr{U}} - \lambda^t \nabla_{\mathbf{x}^t}(\sum_{n\in\mathbb{N}_m} x_n^t - \mathbb{G}_m^t) = 0 \quad (18)$$

where $\mathbf{x}^t = \{x_1^t, x_2^t, \cdots, x_n^t, \cdots, x_{|\mathbb{N}_m|}^t\}$, and $\lambda^t = \{\lambda_1^t, \lambda_2^t, \cdots, \lambda_n^t, \cdots, \lambda_{|\mathbb{N}_m|}^t\}$. Therefore, by solving $\nabla_{\mathbf{x}^t}\tilde{\mathscr{U}}$, we get,

$$\mathscr{K} = \nabla_{\mathbf{x}^t}\tilde{\mathscr{U}} = [e_1 - \gamma_1^t x_1^t - p_m^t; \cdots; e_{|\mathbb{N}_m|} - \gamma_{|\mathbb{N}_m|}^t x_{|\mathbb{N}_m|}^t - p_m^t]^T \quad (19)$$

Now, we find the Jacobi matrix of $\mathscr{K}$ as follows,

$$\mathbf{J}\mathscr{K} = \begin{bmatrix} -\gamma_1^t & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\gamma_{|\mathbb{N}_m|}^t \end{bmatrix} \quad (20)$$

As the Jacobian of $\mathscr{K}$ is a diagonal matrix, and all the elements in the diagonal are negative, we can infer that $\mathbf{x}^t$ has a unique solution, i.e., x_n^t is unique, where $n \in \mathbb{N}_m$. Therefore, there exists a variational equality and GNE solutions. ■

In the proposed DEMANDS algorithm, we also need to determine the existence of GNE for $\mathbb{M}$ micro-grids. Therefore, we try to determine the variational equality for this proposed non-cooperative game, as discussed in Theorem 2.

Theorem 2. *For time slot t, if the set of requested energy $\mathbf{x}_n$, is fixed, there exists a variational equality for utility function $\mathscr{B}_m(p_m^t, x_n^t(p_m^t))$, where $\forall n$, and $\forall m$.*

Proof: We formulate the overall utility function $\tilde{\mathscr{B}}$ of $\mathbb{M}$ micro-grids in a coalition, while satisfying the inequality – $p_m^t \geq c_m^t$, as follows:

$$\tilde{\mathscr{B}}(p_1^t, \cdots, p_{\mathbb{M}}^t; x_1^t, \cdots, x_{|\mathbb{N}|}^t) = \sum_{m=1}^{m\in\mathbb{M}}\sum_{n=1}^{n\in\mathbb{N}_m} p_m^t x_n^t, \quad (21)$$

where c_m is generation cost per unit energy for micro-grid m. Therefore, we need to maximize the overall utility function $\tilde{\mathscr{B}}$ to reach the variational equilibrium solution. Mathematically,

$$\arg\max_{m\in\mathbb{M}} \sum_{m=1}^{m\in\mathbb{M}}\sum_{n=1}^{n\in\mathbb{N}_m} p_m^t x_n^t, \quad (22)$$

Taking help of Lagrangian multiplier, we apply Karush-Kuhn-Tucker (KKT) condition on the utility function $\mathscr{B}_m(p_m^t, x_n^t(p_m^t))$ of each micro-grid m. Therefore,

$$\nabla_{p_m^t}\mathscr{B}_m(p_m^t, x_n^t(p_m^t)) - \varphi_m^t \nabla_{p_m^t}[p_m^t - c_m^t] = 0, \quad (23)$$

$$\varphi_m^t \nabla_{p_m^t}[p_m^t - c_m^t] = 0 \quad (24)$$

where $\varphi_m^t \geq 0$, and φ_m^t is defined as the Lagrangian multiplier for micro-grid m at time slot t. If we perform similar transformation for other micro-grids also, we get the equation for the overall utility function as follows:

$$\nabla_{\mathbf{p}^t}\tilde{\mathscr{B}} - \varphi^t \nabla_{\mathbf{p}^t}[\mathbf{p}^t - \mathbf{c}^t] = 0 \quad (25)$$

where $\mathbf{p}^t = \{p_1^t, p_2^t, \cdots, p_m^t, \cdots, p_{|\mathbb{M}|}^t\}$, and $\mathbf{c}^t =$

$\{c_1^t, c_2^t, \cdots, c_m^t, \cdots, c_{|\mathbb{M}|}^t\}$. Hence, we get,

$$\mathscr{L} = \nabla_{\mathbf{p}^t} \tilde{\mathscr{B}} = [x_1^t, x_2^t, \cdots, x_m^t, \cdots, x_{|\mathbb{M}|}^t]^T \tag{26}$$

where $[\cdot]^T$ defines a transpose matrix of the solution. Therefore, the Jacobian of matrix $\mathscr{L}$ is a *zero matrix*. As $\mathbf{J}\mathscr{L}$ is a zero matrix, we can conclude that for a fixed set of energy requests by $\mathbb{N}_m$ customers, the micro-grid m will have a variational equality point for the price per unit amount of energy, p_m^t, for each time slot t. ■

Therefore, from Theorems 1 and 2, we conclude that the proposed scheme, DEMANDS, has GNE solution for the requested energy by each customer, and the price per unit amount of energy decided by each micro-grid within a coalition.

D. The Proposed algorithm

In this section, we formulate the GNE problem among the customers as a variational inequality problem, and proposed an algorithm that leads to optimally social variational equality solution, which, in turn, leads to GNE solutions. Additionally, the proposed scheme, DEMANDS, is not concerned with which type of sources are there with the micro-grids, as the proposed scheme is only concerned about the amount of energy generated by the micro-grids.

1) Requested Energy optimization: For a fixed price decided by the micro-grids, each customer n decides the amount of energy, x_n^t, to be requested to the micro-grid using Algorithm 1. In such situation, the proposed scheme, DEMANDS, performs well. Applying Karush-Kuhn-Tucker (KKT) condition, we get,

$$\frac{\partial \mathscr{U}_n}{\partial x_n^t} = 0 \Rightarrow e_n - \gamma_n^t x_n^t - p_m^t - \lambda^t = 0 \tag{27}$$

where $\lambda^t \triangleq \lambda_n^t, \quad \forall n$, satisfying the condition in Equation (2). Therefore, the marginal condition is as follows:

$$\mathbb{G}_m^t = \sum_{n \in \mathbb{N}_m} x_n^t \tag{28}$$

However, $\lambda^t(\mathbb{G}_m^t - \sum\limits_{n \in \mathbb{N}_m} x_n^t) = 0$, and $\lambda^t \geq 0$. Therefore,

$$e_n > \gamma_n^t x_n^t + p_m^t \tag{29}$$

Now, for overall utility function,

$$\sum_{n \in \mathbb{N}_m} e_n > \mathbb{G}_m^t \sum_{n \in \mathbb{N}_m} \gamma_n^t + \mathbb{N}_m p_m^t. \tag{30}$$

Equating the condition for variational equality, we get,

$$x_n^{t*} = \frac{e_n - p_m^t - \lambda^t}{\gamma_n^t}, \tag{31}$$

2) Price optimization: Having analyzed the requested energy of the customers, each micro-grid tries to find the optimum price p_m^{t*} using Algorithm 2. Hence, the dynamic energy request is taken into account in the proposed scheme, DEMANDS, while using dynamic pricing mechanism. From Equation (16), we infer that,

$$p_m^t \leq e_n - \gamma_n^t x_n^t \tag{32}$$

Algorithm 1: DEMANDS algorithm for each customer

Input: Each micro-grid m decides the optimum price per unit energy, p_m^{t*}.
Output: Each customer n decides the amount of energy to be requested to micro-grid m.
if $\mathscr{U}_n(\tilde{x}_n^t, x_{-n}^{t*}, p_m^{t*}) \ngeq \mathscr{U}_n(x_n^t, x_{-n}^{t*}, p_m^{t*})$ **then**
  Evaluate λ_n^t solving equation, $\lambda_n^t = e_n - \gamma_n^t x_n^t - p_m^{t*}$;
  $\tilde{x}_n^t$ is the modified amount of requested energy;
  Evaluate $\tilde{x}_n^t$ by solving the following equation,
  $\tilde{x}_n^t = \frac{e_n - p_m^t - \lambda_n^t}{\gamma_n^t}$;
  Send the request message with the $\tilde{x}_n^t$ amount of requested energy;
else
  The amount of energy to be requested is fixed;
  The *Fixed Flag* in the message of the customer n is set;

Algorithm 2: DEMANDS algorithm for each micro-grid

Input: Each customer n decides the amount of energy to be requested, x_n^t, to micro-grid m.
Output: Each micro-grid m decides the optimum price per unit energy, p_m^t.
if $\mathscr{B}_m(\tilde{p}_m^t, x_n^{t*}(\tilde{p}_m^t)) \ngeq \mathscr{B}_m(p_m^t, x_n^{t*}(p_m^t))$ **then**
  Evaluate $\tilde{p}_m^t$ using the following equation:
  $\tilde{p}_m^t = \frac{\sum\limits_{n \in \mathbb{N}_m} e_n - \sum\limits_{n \in \mathbb{N}_m} \gamma_n^t x_n^{t*}}{|\mathbb{N}_m|}$;
  Send the reply message with the $\tilde{p}_m^t$ price per unit energy information;

Considering that the requested energy by customer n is optimal, i.e., $x_n^t = x_n^{t*}$, we modify Equation (32) as follows:

$$p_m^t \leq e_n - \gamma_n^t x_n^{t*} \tag{33}$$

Therefore, for micro-grid m, the maximum price per unit energy, i.e., the decided optimum price, p_m^{t*}, is as follows:

$$p_m^{t*} = e_n - \gamma_n^t x_n^{t*} \tag{34}$$

Hence, each micro-grid m chooses the optimum price per unit energy based on the total requested energy, $\sum\limits_n x_n^t$, by $\mathbb{N}_m$ customers.

V. Performance Evaluation

A. Simulation Settings

We considered randomly generated positions for the micro-grids and the customers on a MATLAB-based simulation platform. We considered that the micro-grids form a coalition based on their geographical location, as discussed in [9]. In this work, we have taken randomly generated values for the amount of requested energy for each customer and the amount of generated energy by each micro-grid at each time slot, as shown in Table 1. Therefore, we claim that the proposed scheme, DEMANDS, is able to handle the randomness in the energy generation by the micro-grids. We considered that the total required energy of a customer is fixed in a day. Based on the requested energy by all the customers connected with the micro-grid, the micro-grid decides the optimum price per unit energy based the variational equilibrium solution.

We have evaluated the performance of the proposed algorithm, DEMANDS, by comparing with the online optimal real-time energy distribution algorithm (OORA) [29], and the Optimal Real-time Pricing Algorithm (ORPA) [30] through simulations. In OORA, the authors proposed an algorithm for energy distribution with real-time pricing scheme. However, the OORA algorithm is not based on game theory. On the other hand, Samadi *et al.* [30] proposed a real-time pricing algorithm having an energy consumption controller (ECC), which is

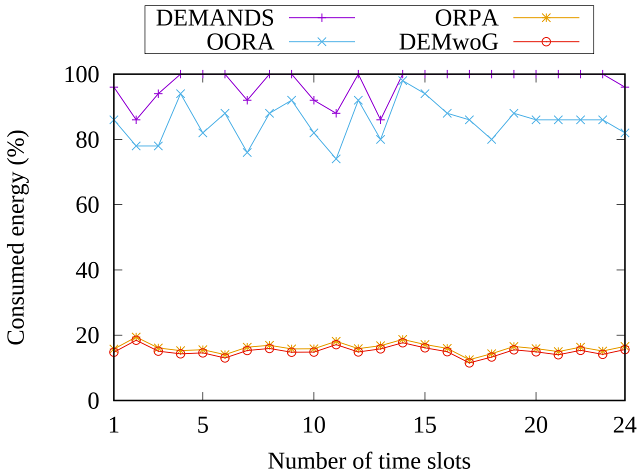

Fig. 4: Consumed energy

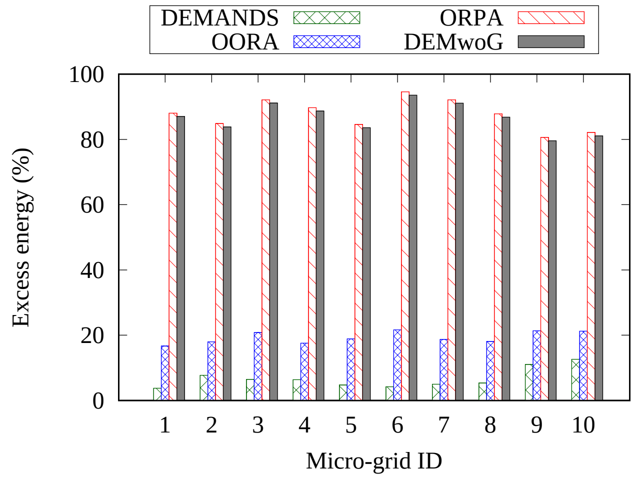

Fig. 5: Excessed energy

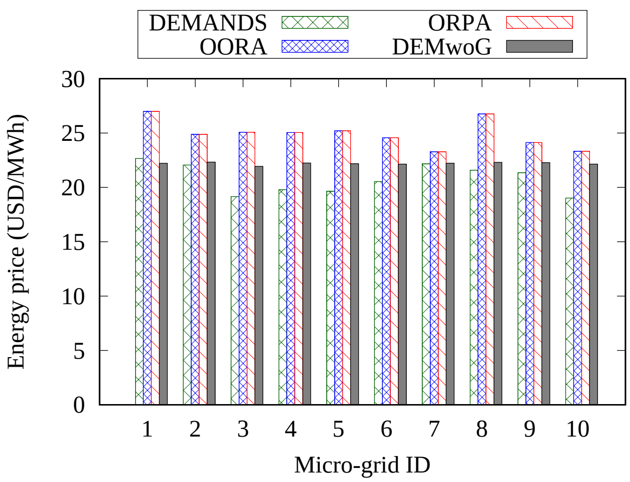

Fig. 6: Price per unit energy

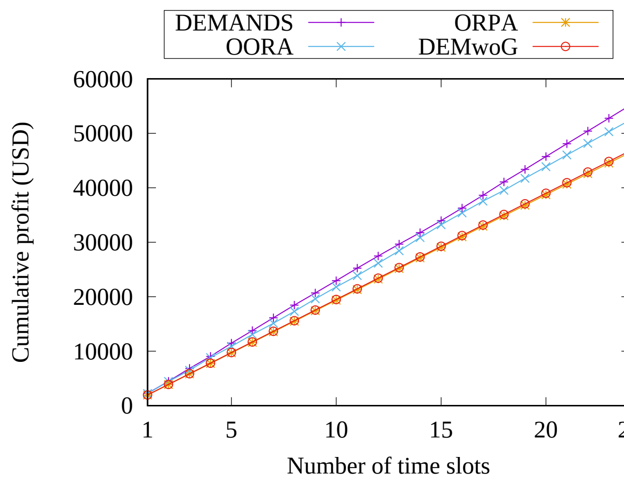

Fig. 7: Cumulative profit

based on a microeconomics approach. However, they did not consider noon-cooperative scheduling. Thus, we endeavored to improve the performance of energy scheduling using the proposed DEMANDS algorithm. In addition to OORA and ORPA, we also simulated another scheme similar to DEMANDS, but does not use game theory. We named this scheme as distributed energy management without game (DEMwoG).

B. Performance Metrics

Requested energy by customers: In a coalition, each customer tries to maximize his/her utility by maximizing the requested energy to the micro-grid with lower price.

Excess energy of micro-grids: Each micro-grid tries to reduce the unused generated energy, i.e., excess energy, by choosing an optimum price per unit energy.

Price per unit energy: The micro-grids choose the optimum price per unit energy such that other customers are motivated to request energy, and the micro-grid utilizes the generated energy properly in each time slot.

Percentage of energy service served: We evaluate the overall performance with the percentage of customers served. If a customer gets $r\%$ of the initially requested energy, s/he gets $r\%$ of energy service.

C. Results and Discussions

For simulation, we assume that each micro-grid monitors the real-time supply and demand in every 5 seconds interval. After every 5 second, each micro-grid has to check the demand-supply curve, and if there is any modification in the demand curve. However, one micro-grid may also monitor the supply-demand curve, continuously, which will be computationally expensive. Hence, we chose 5 second interval to reduce the computational complexity, and took advantage of having energy generation and demand side management in smart grid.

In Figure 4, the percentage of total consumed energy in each time slot within a coalition is shown. Figure 4 shows that the energy consumed in each time slot is within $90\% - 100\%$ for DEMANDS, whereas the consumed energy is within $75\% - 90\%$ for OORA, and $15\% - 20\%$ for ORPA. Therefore, we can infer that the satisfaction level of the customers is almost 9.6%, and 80% higher using DEMANDS than using OORA, and ORPA, respectively. Figure 4 shows that the excess amount of energy is much less in DEMANDS than OORA,

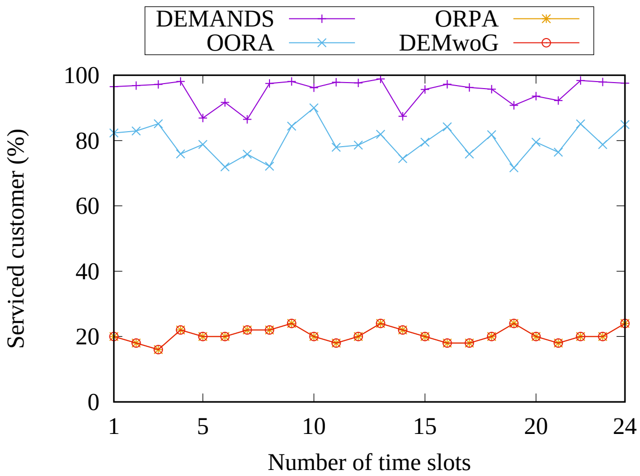

Fig. 8: Customers served

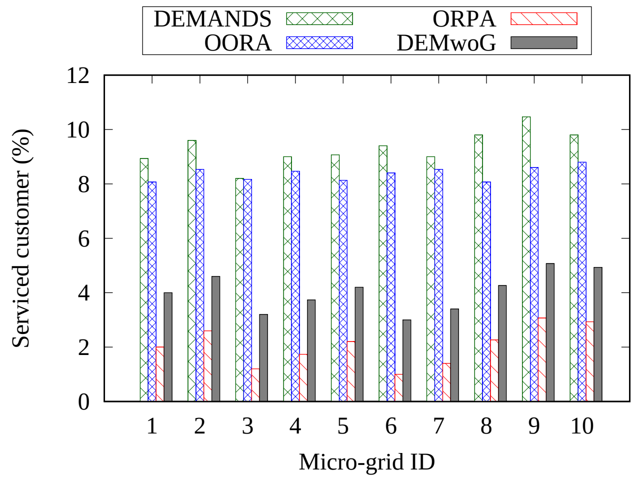

Fig. 9: Customers served

TABLE I: Simulation Parameters

Parameter	Value
Simulation area	$20{\times}20\ km^2$
Number of micro-grids	10
Number of Customers	500
Customer's minimum requested energy	65 MWh
Customer's maximum requested energy	110 MWh
Micro-grid's minimum generated energy	500 MWh
Micro-grid's maximum generated energy	750 MWh
Generation cost per MWh energy	10-20 USD

ORPA, and DEMwoG. From Figure 4, we conclude that the underutilization of generated energy is 16.67%, 76.75%, 77% higher than DEMANDS in OORA, ORPA, and DEMwoG, respectively. Figure 5 reestablishes that the utilization of generated energy is much higher using DEMANDS than using OORA, ORPA, and DEMwoG. The excess amount of generated energy is 50%-66% higher than DEMANDS, for OORA, ORPA, and DEMwoG. Figure 6 shows that the price per unit energy is much lower for DEMANDS than OORA, ORPA, and DEMwoG. Therefore, the customers get the required energy with much lower price. Additionally, we consider that the micro-grids sell the excess amount of generated energy to the main grid. We consider that the price per unit energy paid by the main grid is constant, i.e., the minimum selling price to the customers, for different schemes, such as DEMANDS, OORA, ORPA, or DEMwoG. Hence, we get that using DEMANDS, the profit of each micro-grid is 6.76%, 19.03%, and 18.24% higher than using OORA, ORPA and DEMwoG, respectively, as shown in Figure 7. Figure 8 shows that using the proposed DEMANDS scheme, almost 100% request of each customer is served. However, the percentage of customers served in each time-slot is within 70%-95%, and 15%-25% using OORA, and ORPA, respectively. Figure 9 shows that, on an average the percentage of customers served by each micro-grid in a day is also higher in DEMANDS than OORA, ORPA, and DEMwoG. In Figure 9, the percentage of customers served by each micro-grid remains almost invariant. However, the percentage of customers served by each micro-grid is much lower using ORPA than using DEMANDS. Therefore, we conclude that the energy requested by the customers are more distributed among the available micro-grids using dynamic price model, and the generated energy is uniformly utilized using DEMANDS than using OORA, ORPA, and DEMwoG. Hence, in addition to ensuring maximum use of renewable energy sources and the minimal use of tradition energy resources, the proposed scheme, DEMANDS, also ensures proper load balancing among the available micro-grids.

VI. Conclusion

In this paper, we formulated a multi-leader multi-follower Stackelberg game to study the problem of distribution energy management using scheduling. Based on the proposed algorithm, DEMANDS, we showed how a customer decides the optimum amount of energy to be requested, when each customer is connected with multiple micro-grids. The micro-grids also choose an optimum price per unit energy, so as to maximize its profit, and utilize its generated energy properly. The simulation results show that the proposed approach yields improved results.

Future extension of this work includes understanding how the generated energy can be distributed using a centralized management unit. This work can also be extended by introducing plug-in hybrid electric vehicles in a smart grid. Additionally, this work can be extended by considering the uncertainty in energy generation by the micro-grids and the uncertainty in customers' energy consumption.

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