Game-Theoretic Energy Trading Network Topology Control for Electric Vehicles in Mobile Smart Grid

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Abstract

Existing works on energy trading consider different schemes for forming energy trading networks, which assume that each plug-in hybrid electric vehicle (PHEV) is connected with a single micro-grid. Consequently, in on-peak hours, a PHEV gets the requested energy during the allotted time slot by paying a higher price. Alternatively, the PHEV waits for a significant duration of time to get serviced until the on-peak hour elapses. In this paper, we propose that a PHEV may get energy from any of the available micro-grids within a coalition instantaneously without paying higher price. In this work, the problem of energy trading network topology control for PHEVs is studied as a *multi-leader multi-follower Stackelberg game*. In this game, each PHEV acts as a leader, and decides the amount of energy to be requested to the selected micro-grid. On the other hand, the micro-grids act as followers, need to decide the price per unit energy. Using variational inequality, it is shown that the proposed scheme, *energy trading network topology control* (ENTRANT), has generalized Nash equilibrium, which is also socially optimal. ENTRANT enables the PHEVs and the micro-grids within a coalition to reach the equilibrium state, is evaluated theoretically, as well as through simulations.

Keywords

Micro-grid, Plug-in Hybrid Electric Vehicles, Energy Trading, Stackelberg Game, Energy Network Topology Control, Mobile Smart Grid.

I. Introduction

The traditional electrical grids with advanced techniques, termed as *smart grid*, are envisioned to achieve high reliability of the energy trading networks. A smart grid [1]–[3] is visualized to be a cyber-physical system, which is highly efficient and robust in nature, equipped with sustainable models of energy production, distribution, and usage. Smart grid integrates several advanced mechanisms such as use of advanced metering infrastructure (AMI) [4], automated meter reading (AMR) [4], smart meter (SM), data aggregation unit (DAU), meter data management system (MDMS), distributed energy resources (DER), energy management systems (EMS) [5], intelligent electronic devices (IEDs), and plug-in hybrid electrical vehicles (PHEVs) [6], in an electrical grid. In traditional energy trading networks, the energy is distributed unidirectionally to the residential customers by the centralized main grid, and the customers pay an amount decided by the energy service provider after a fixed duration. However, in smart grid having duplex communication infrastructure, the modernized energy trading network has bidirectional electricity exchange facility. The large-scale energy trading network in a smart grid is divided into several micro-grids, which can either supply energy directly to the customers and the PHEVs, or exchange energy with the main grid through the substations. In the presence of several micro-grids, it is desired that each micro-grid serves a small geographical area, i.e., a group of customers, to relax the load on the main grid. One of the important features in smart grid is distributed energy management, in which each customer or PHEV can demand energy according to its requirement, and also can schedule appliances depending on his/her/its preferences.

A. Motivation

In a smart grid, the concept of green energy generation using renewable-energy resources, i.e., reducing CO_2 emission in the environment for energy generation, is prioritized. The micro-grids generate energy typically based on renewable-energy resources. Therefore, the amount of energy generated by each micro-grid is not fixed for each time-slot. As a result, the customers and the PHEVs, i.e., consumers, either have to pay higher price per unit energy or will have to wait for a certain time duration. If the energy requested by the consumers is higher than the amount of energy generated, the consumers can get energy by paying high. In that case, the micro-grid requests other micro-grids having excess amount of energy to serve the demand. Otherwise, the consumers wait for a certain time duration to get the energy-service, while paying less. On the other hand, if the micro-grids have excess amount of generated energy, it can supply that excess amount either to the other micro-grids having demand of energy or to the main grid through the substations. Therefore, the energy loss through transmission line is higher. Moreover, according to the existing literature, the PHEVs are connected with a single micro-grid. Therefore, each PHEV has to either pay high amount per unit energy or has to wait for a certain time duration to get the energy services. Therefore, we propose an *energy trading network topology control* scheme for PHEVs. In this scheme, we consider that each PHEV has the option to request the amount of required energy to any of the available micro-grids. Hence, the PHEVs do not have to wait to get the energy service. On the other hand, to get the required amount of energy, the PHEVs do not have to pay high. Therefore, using the proposed energy trading network topology control scheme, the PHEVs get higher satisfaction by consuming the required amount of energy, while the available micro-grids in the coalition ensure better quality of energy service (QoS). Hence, using the proposed scheme, the loss through the transmission line gets reduced, and the micro-grids having excess energy get higher revenue by supplying the excess amount of generated energy.

B. Contribution

In this paper, we propose a non-cooperative *energy trading network topology control* (ENTRANT) scheme for mobile PHEVs in smart grid. We use a multi-leader multi-follower non-cooperative Stackelberg game [7] in the proposed scheme, ENTRANT, to estimate the amount of energy to be consumed by the PHEVs and price per unit energy to be charged by the micro-grids. Using ENTRANT, the PHEVs consume higher amount energy, i.e., the satisfaction factor after consuming energy is high, while paying less price per unit energy. On the other hand, the micro-grids utilize the amount of generated energy properly, while ensuring higher revenue by selling the excess amount of generated energy to the PHEVs after satisfying the demand of the customers. We summarize the *contributions* of this paper as follows:

(i) We propose a game-theoretic energy trading network topology control scheme, i.e., ENTRANT, for proper distribution of generated energy, while ensuring the revenue maximization of the micro-grids.

(ii) We formulate the interaction between the micro-grids and the PHEVs as a multi-leader multi-follower Stackelberg game. This game model establishes the broader aspect for energy trading network in mobile smart grid.

(iii) We present an algorithm for maintaining the energy trading network in mobile smart grid using the proposed scheme, ENTRANT. The PHEVs decide the amount of energy to be consumed based on the amount of energy required for satisfying the requirement. On the other hand, the micro-grids decide the price per unit energy, while ensuring the proper utilization of generated energy, and the revenue by supplying the requested amount of energy by the PHEVs.

The remainder of the paper is organized as follows. Section II summarizes the related work in the area of energy management in smart grid. The system model is discussed in Section III. The proposed scheme, ENTRANT, is described in Section IV. Section V depicts the simulation setup and performance evaluation of the proposed scheme, ENTRANT, considering different performance metrics. Finally, Section VI concludes the paper by citing directions for future work.

II. Related Work

In the last few years, lot of research work on smart grid emerged, viz., [8]–[28]. Some of the existing literature are discussed in this Section. Saad *et al.* [8] formulated a coalition game having multiple micro-grids, and proposed a distributed algorithm for forming the coalition. The authors [8] assumed that one micro-grid can exchange excess energy with the main grid. They did not consider any scheduling approach for distributing the generated energy properly, while satisfying the energy demand of the customers. In case of power exchange between the micro-grid and the main grid, there may be loss of energy over the distribution line. They formulated the utility of micro-grid

i by taking into account the loss of energy, as follows [8]:

$$U(i) = -w_i P_{i0}^{loss} \tag{1}$$

Misra *et al.* [10] suggested a distributed dynamic pricing mechanism (D2P) for charging PHEVs. They used two different pricing schemes namely home pricing and roaming pricing. Such and Hill [12] proposed that efficient and economic operation of an electric energy distribution system can be improved with the implementation of wind generation and storage devices. In such a scenario, they proposed that, if in a certain geographical area, there are some storage devices and some wind generation, which are controllable by the micro-grid, the micro-grid decides whether or not to use these. The rate of variation of wind power is also controlled to have smooth energy supply to the customers. Mondal and Misra [14] proposed a decision making process to form coalitions dynamically between micro-grids and customers.

Bakker *et al.* [15] recommended a distributed load management scheme with dynamic pricing, and have modeled it as a network congestion game. Nash equilibrium is presented in order to have an optimal solution. However, in this scenario, if the customer requires more energy in a certain time, then s/he has to wait for some time until the distributed generation can manage it, as the customer does not have the option to switch to another micro-grid, and fulfill his/her requirement from that micro-grid. Molderink *et al.* [16] proposed an algorithm by using the energy in the off-peak, and the on-peak hours, with a virtual power plant, for energy management. Additionally, they showed that the renewable energy sources are useful to achieve cost effective and environment-friendly energy supply to the end users.

Vytelingum *et al.* [17], [18] proposed an algorithm, in which the customers choose their strategies based on their advance knowledge about the market. The authors discussed about storage devices and benefits from micro-storage implementation. Sanseverino *et al.* [19] studied an algorithm for load shifting and storage device management. The authors proposed that during peak-hour, heavy loads should be turned off, and vice-versa. They compared the control mechanism with real storage devices to show the impact of the load shifting scheme on the smart grid. However, in this scenario, if the customer does not want any delay in receiving the requested energy, s/he has to pay high price for it. Fang *et al.* [23] proposed different energy management schemes. However, in their work, new opportunities for improved residential energy management and bill reduction are studied without considering the impact of stochastic scheduling approach for distributed energy management. Erol-Kantarci and Mouftah [29] proposed a time-to-use (TOU) aware-energy management scheme. In their proposed scheme, a customer consumes energy according to the time, whether it is an on-peak hour or an off-peak hour. If it is an on-peak hour, the customer waits for being served. Otherwise, the customer demands the required energy without waiting, if the delay is greater than the maximum allowable delay which is a local variable to the customer. Yet, the energy management policy adopted by the customers and the micro-grids need further research to have an optimal solution and with minimum delay and less message overhead.

In contrast to the existing work, a game theoretic energy trading network topology control scheme is proposed for the PHEVs functioning in a mobile smart grid environment. We use a non-cooperative multi-leader multi-follower Stackelberg game theoretic approach to develop an optimal solution of energy trading network topology in mobile smart grid.

III. System Model

We consider an energy trading network in smart grid consisting of multiple micro-grids and several *consumers*. The consumers, i.e., the customers and the plug-in hybrid vehicles (PHEVs), may request for energy service to any micro-grid from the set of available micro-grids in a coalition [14]. After meeting the energy requirements of the customers connected to the micro-grids, each micro-grid decides to sell the excess amount of generated energy to the PHEVs, which are available in the coalition at that time instant in mobile smart grid. The schematic diagram of the energy trading network for mobile smart grid environment is shown in Figure 1. We consider that each micro-grid has a communicating device, i.e., meter data management system (MDMS). With the help of MDMS, each micro-grid sends information to the data aggregation points (DAPs) using wide area network (WAN). Each DAP communicates with the smart meters, which are associated with the consumers, i.e., the customers and the PHEVs, using neighborhood area network (NAN). The appliances at the consumer-end send their energy-consumption informations to the smart meters using home area network (HAN).

We consider that at each time slot $t \in \mathcal{T}$, where $\mathcal{T}$ is the set of time slots in a day, in a coalition, each consumer $n \in \mathcal{N}(t)$, where $\mathcal{N}(t)$ is the set of the consumers at time slot t, consumes $d_n(t)$ amount of energy. At each time slot $t \in \mathcal{T}$, the set of consumers, $\mathcal{N}(t)$, is combination of the set of customers ,$\mathcal{N}^c(t)$, who are static in nature, and the set of PHEVs ,$\mathcal{N}^p(t)$, which are mobile in nature. Mathematically,

$$\mathcal{N}(\cdot) = \mathcal{N}^c(\cdot) \cup \mathcal{N}^p(\cdot) \tag{2}$$

We consider that at time slot $t \in \mathcal{T}$, each micro-grid $m \in \mathcal{M}$, where $\mathcal{M}$ is the set of micro-grids in the coalition, generates $\mathcal{G}_m(t)$ amount of energy. After meeting the energy demand of the set of connected customers, $\mathcal{N}_m^c(t)$, i.e., $\mathcal{C}_m(t)$, each micro-grid m has $\mathcal{E}x_m(t)$ amount of excess energy, as shown below:

$$\mathcal{E}x_m(\cdot) = \mathcal{G}_m(\cdot) - \mathcal{C}_m(\cdot) \tag{3}$$

where $\mathcal{C}_m(\cdot) = \sum_{n \in \mathcal{N}_m^c(\cdot)} d_n^c(\cdot)$, $\mathcal{N}_m^c(\cdot) \subseteq \mathcal{N}^c(\cdot)$, and $d_c^{(n)}(\cdot)$ is the amount of energy requested by each customer $n \in \mathcal{N}_m^c(\cdot)$. The excess amount of generated energy by each micro-grid m, i.e., $\mathcal{E}x_m(\cdot)$, must satisfy the following

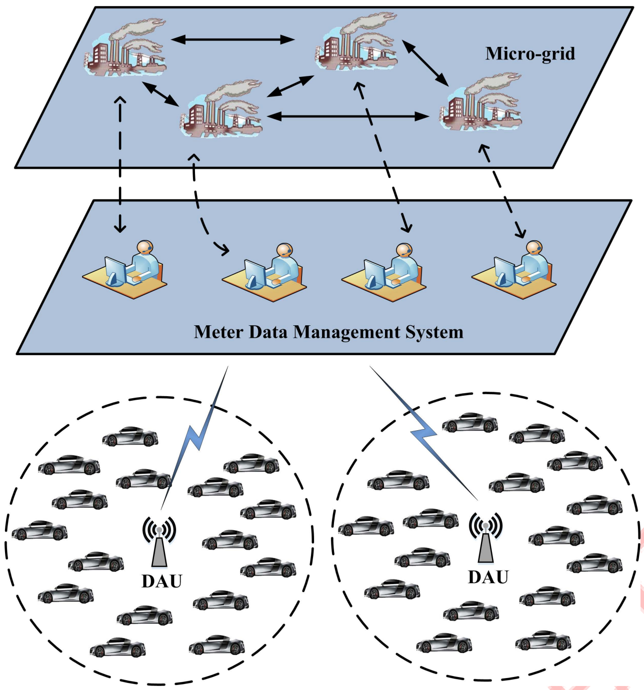

Fig. 1: Schematic diagram of energy trading network

constraint:

$$\mathcal{E}x_m(\cdot) \geq 0, \quad \forall m \in \mathcal{M} \tag{4}$$

The amount of excess energy generated by each micro-grid $m \in \mathcal{M}$, i.e., $\mathcal{E}x_m(\cdot)$, is used for serving a subset of the available PHEVs, $\mathcal{N}^p(\cdot)$, i.e., $\mathcal{N}^p_m(\cdot)$, in the coalition. Mathematically,

$$\mathcal{N}^p_m(\cdot) \subseteq \mathcal{N}^p(\cdot) \tag{5}$$

Each PHEV $n \in \mathcal{N}^p(\cdot)$ has a requirement of d^p_n amount of energy. To fulfill the energy requirement, each PHEV n chooses a micro-grid m having $\mathcal{E}x_m(\cdot)$ amount of generated energy. Therefore, the total amount of energy requested to each micro-grid m by the PHEVs $\mathcal{N}^p_m(\cdot)$, where $\mathcal{N}^p_m(\cdot)$ is the set of PHEVs demanded energy from micro-grid $m \in \mathcal{M}$ and $\mathcal{N}^p_m(\cdot) \subseteq \mathcal{N}^p(\cdot)$, must satisfy the following constraint:

$$\mathcal{E}x_m(\cdot) \geq \sum_{n=1}^{n \in \mathcal{N}^p_m(\cdot)} d^p_n(\cdot) \tag{6}$$

On the other hand, based on the total demanded amount of energy, each micro-grid $m \in \mathcal{M}$ decides the *price coefficient*, $p_m(\cdot)$, i.e., the multiplying factor of the price per unit energy to be paid by the set of connected PHEVs, $\mathcal{N}^p_m(\cdot)$. The price coefficient of each micro-grid m, i.e., $p_m(\cdot)$, is defined in Definition 1.

Definition 1. *The price coefficient of each micro-grid* m*, i.e.,* $p_m(\cdot)$*, is a function of the ratio of the amount of energy requested to micro-grid* m*, i.e.,* $\sum_{n=1}^{n\in\mathcal{N}_m^p(\cdot)} d_n^p(\cdot)$*, and the excess amount of generated energy by micro-grid* m*, i.e.,* $\mathcal{E}x_m(\cdot)$*. Mathematically,*

$$\begin{aligned} p_m(\cdot) &= f\left(\sum_{n=1}^{n\in\mathcal{N}_m^p(\cdot)} \tilde{d}_n^p(\cdot), \mathcal{E}x_m(\cdot)\right) \\ &= \frac{\sum\limits_{n=1}^{n\in\mathcal{N}_m^p(\cdot)} \tilde{d}_n^p(\cdot)}{\mathcal{E}x_m(\cdot)} \end{aligned} \tag{7}$$

With the increase in the number of connected PHEVs with micro-grid m, the amount of energy requested to micro-grid m increases. Hence, the price coefficient increases. On the other hand, the quality of energy service provided by the micro-grids decreases with the increase in the amount of requested energy by the PHEVs. The *satisfaction factor* of the micro-grid m, i.e., $s_m(\cdot)$, also increases with the decrease in the remaining amount of generated energy. We define the satisfaction factor of micro-grid $m \in \mathcal{M}$ in Definition 2.

Definition 2. *The satisfaction factor of micro-grid* $m \in \mathcal{M}$*,* $s_m(\cdot)$*, is defined by the ratio of remaining amount of excess energy, i.e.,* $\mathcal{E}x_m^{res}(\cdot)$*, and the excess amount of generated energy after fulfilling the energy demand of the customers,* $\mathcal{N}_m^c(\cdot)$*, i.e.,* $\mathcal{E}x_m(\cdot)$*. Mathematically,*

$$\begin{aligned} s_m(\cdot) &= \frac{\mathcal{E}x_m^{res}(\cdot)}{\mathcal{E}x_m(\cdot)} \\ &= \frac{\mathcal{E}x_m(\cdot) - \sum\limits_{n=1}^{n\in\mathcal{N}_m^p(\cdot)} d_n^p(\cdot)}{\mathcal{E}x_m(\cdot)} \\ &= 1 - \frac{\sum\limits_{n=1}^{n\in\mathcal{N}_m^p(\cdot)} d_n^p(\cdot)}{\mathcal{E}x_m(\cdot)} \end{aligned} \tag{8}$$

Mobility Model for Mobile Smart Grid Environment

We assume that the PHEVs follow the mobility pattern of Gauss-Markov mobility model [10], [30]. According to the mobility model, each PHEV n updates its location periodically, after crossing a threshold distance. The position and the velocity are considered to be correlated with time, i.e., the position of each PHEV n at time instant τ depends on the location and velocity of the PHEV at previous time instant $(\tau - 1)$. We consider that the PHEVs move in a two-dimensional plane. Hence, the Gauss-Markov mobility model is represented as [30]:

$$\vec{\upsilon}_n(\tau) = \upsilon_n^x(\tau)\hat{i} + \upsilon_n^y(\tau)\hat{j} \tag{9}$$

where $\upsilon_n^x(\cdot)$ and $\upsilon_n^y(\cdot)$ are the velocity components of each PHEV n towards **x** and **y** direction. We define $\vec{\upsilon}_n(\tau)$ as follows:

$$\vec{\upsilon}_n(\tau) = \alpha\upsilon_n(\tau - 1) + (1 - \alpha)\mu + \sigma\sqrt{1 - \alpha^2}\mathcal{W}(\tau - 1) \tag{10}$$

where $\vec{\upsilon}_n(\tau)$ denotes the velocity vector of PHEV n at time τ, α is the variance over time, μ signifies the mean of the velocity, σ denotes the standard deviation, and $\mathcal{W}(\cdot)$ is an uncorrelated Gaussian process with zero mean with unit variance and is independent. Therefore, we define the components of $\vec{\upsilon}_n(\cdot)$, i.e., $\upsilon_n^x(\tau)$ and $\upsilon_n^y(\tau)$ in Equation (9), as follows:

$$\upsilon_n^x(\tau) = \alpha\upsilon_n^x(\tau - 1) + (1 - \alpha)\mu^x + \sigma^x\sqrt{1 - \alpha^2}\mathcal{W}^x(\tau - 1) \tag{11}$$

$$\upsilon_n^y(\tau) = \alpha\upsilon_n^y(\tau - 1) + (1 - \alpha)\mu^y + \sigma^y\sqrt{1 - \alpha^2}\mathcal{W}^y(\tau - 1) \tag{12}$$

We define the direction of each PHEV n, i.e., $\theta_n(\cdot)$, as follows:

$$\theta_n(\cdot) = \tan^{-1}\left(\frac{\upsilon_n^y(\cdot)}{\upsilon_n^x(\cdot)}\right) \tag{13}$$

To design the mobility model, we consider that α is a constant, and $0 \leq \alpha \leq 1$.

Communication Model for Mobile Smart Grid Environment

We assume that the communication topology between the micro-grids and the PHEVs is a wireless mesh network (WMN). We use the IEEE 802.11b protocol for communication between the micro-grids and the PHEVs. Initially, each PHEV $n \in \mathcal{N}^p(\cdot)$ sends a request message having information of the amount of required energy. The request message format of each PHEVs is shown in Figure 2(a).

ReqMsgType	PHEV_ID	ReqEnergy	FinalSelectFlag
1 byte	4 byte	2 byte	1 byte

(a) Request message format

AckMsgType	MG_ID	Price	AckFlag
1 byte	4 byte	2 byte	1 byte

(b) Reply message format

Fig. 2: Message formats using ENTRANT scheme

Based on the total amount of requested energy, each micro-grid decides the price coefficient, $p_m(\cdot)$, and the price per unit energy to be paid by each PHEV. The reply message format of each micro-grid is shown in Figure 2(b).

IV. An Energy Trading Network Topology Control Game

A. Game Formulation

To study the interaction between the PHEVs and the micro-grids in *energy trading network topology control* (ENTRANT), we use a non-cooperative multi-leader multi-follower Stackelberg game theoretic approach [7]. In ENTRANT, each PHEV acts as a leader, and needs to decide the amount of energy to be requested to the selected micro-grid. The micro-grids are the followers, which decide the price per unit energy based on the amount of requested energy by the leaders, i.e., the PHEVs. The components of the proposed scheme, ENTRANT, are as follows:

i) Each PHEV $n \in \mathcal{N}^p(\cdot)$ selects a micro-grid $\tilde{m}$ from the set of available micro-grids,i.e., $\mathcal{M}$, within the communication range.

ii) Each PHEV $n \in \mathcal{N}^p(\cdot)$ decides the amount of energy to be requested, i.e., $d^p_n(\cdot)$, to the selected micro-grid $\tilde{m} \in \mathcal{M}$, when each micro-grid m has $\mathcal{E}x_m(\cdot)$ amount of excess energy after meeting the requirement of the set of connected customers, i.e., $\mathcal{N}^c(\cdot)$.

iii) Based on the requested amount of energy by the PHEVs, $\mathcal{N}^p_m(\cdot)$, each micro-grid $m \in \mathcal{M}$ decides the price coefficient $p_m(\cdot)$ using Equation (7).

1) Utility function of a PHEV: In the proposed scheme, ENTRANT, the utility function of each PHEV $n \in \mathcal{N}^p(\cdot)$, i.e., $\mathbb{U}^{\tilde{m}}_n(\cdot)$, represents the satisfaction factor PHEV n by consuming $d^p_n(\cdot)$ amount of energy from micro-grid $\tilde{m}$. The satisfaction factor of each PHEV $n \in \mathcal{N}^p(\cdot)$ is defined in Definition 3.

Definition 3. *The satisfaction factor of each PHEV $n \in \mathcal{N}^p(\cdot)$ is evaluated with the ratio of the amount of energy requested to the selected micro-grid $\tilde{m} \in \mathcal{M}$, i.e., $d^p_n(\cdot)$, and the maximum amount of energy required, i.e., $\max d^p_n(\cdot)$ defined as below:*

$$\max d^p_n(\cdot) = E^{max}_n - E^{res}_n(\cdot) \tag{14}$$

where E^{max}_n and $E^{res}_n(\cdot)$ are the maximum battery capacity and the residual energy of each PHEV $n \in \mathcal{N}^p(\cdot)$, respectively.

We define the rules for utility calculation of each PHEV n as follows:

i) The utility function of each PHEV n, $\mathbb{U}^{\tilde{m}}_n(\cdot)$, is considered to be a non-decreasing function. Hence, in each time-slot, with the increase in the amount of consumed energy $d^p_n(\cdot)$, i.e., $\bar{d}^p_n(\cdot) = \tilde{d}^p_n(\cdot) - d^p_n(\cdot)$, the satisfaction factor of each PHEV n becomes higher. Here, $\tilde{d}^p_n(\cdot)$ and $d^p_n(\cdot)$ are the new and the modified recent amount of requested energy by PHEV n to the selected micro-grid $\tilde{m} \in \mathcal{M}$. Mathematically,

$$\frac{\delta \mathbb{U}_n^{\tilde{m}}(\cdot)}{\delta \tilde{d}_n^p(\cdot)} \geq 0 \tag{15}$$

ii) The marginal utility of each PHEV n is considered to be decreasing, as with increase in consumed energy after reaching equilibrium state, the PHEVs will be over powered or the PHEVs have to pay a huge amount. Mathematically,

$$\frac{\delta^2 \mathbb{U}_n^{\tilde{m}}(\cdot)}{\delta [\tilde{d}_n^p(\cdot)]^2} < 0 \tag{16}$$

iii) The amount of energy to be consumed reduces with the increase in price coefficient. Therefore, the utility value of $\mathbb{U}_n^{\tilde{m}}(\cdot)$ reduces with the increase in price coefficient of the selected micro-grid $\tilde{m} \in \mathcal{M}$. Mathematically,

$$\frac{\delta \mathbb{U}_n^{\tilde{m}}(\cdot)}{\delta p_{\tilde{m}}(\cdot)} < 0 \tag{17}$$

Therefore, for each PHEV $n \in \mathcal{N}^p(\cdot)$, we define the revenue function, $\mathbb{R}_n^p(\cdot)$, and the cost function, $\mathbb{C}_n^p(\cdot)$, in Definitions 4 and 5, respectively. We consider that the utility function, $\mathbb{U}_n^{\tilde{m}}(\cdot)$, of each PHEV n is defined as the difference of revenue function, $\mathbb{R}_n^p(\cdot)$, and the cost function, $\mathbb{C}_n^p(\cdot)$. Mathematically,

$$\mathbb{U}_n^{\tilde{m}}(\cdot) = \mathbb{R}_n^p(\cdot) - \mathbb{C}_n^p(\cdot) \tag{18}$$

Definition 4. *The revenue function of each PHEV n, i.e., $\mathbb{R}_n^p(\cdot)$, is considered to be a concave function. Therefore, we define the revenue function, $\mathbb{R}_n^p(\cdot)$, as follows:*

$$\begin{aligned} \mathbb{R}_n^p(\cdot) &= E_n^{max} \tan^{-1}\left(e^{-\frac{\bar{d}_n^p(\cdot)}{\tilde{d}_n^p(\cdot)}}\right) \\ &= E_n^{max} \tan^{-1}\left(e^{-\frac{\bar{d}_n^p(\cdot) - d_n^p(\cdot)}{\tilde{d}_n^p(\cdot)}}\right) \end{aligned} \tag{19}$$

Definition 5. *The cost function of PHEV $n \in \mathcal{N}^p(\cdot)$, i.e., $\mathbb{C}_n^p(\cdot)$, is considered to be a linear function having linear coefficient of the selected micro-grid $\tilde{m}$, i.e., price coefficient defined in Equation (7). Mathematically,*

$$\begin{aligned}\mathbb{C}_n^p(\cdot) &= p_{\tilde{m}}(\cdot)\tilde{d}_n^p(\cdot)\\ &= f\left(\frac{\sum_{n=1}^{n\in\mathcal{N}_m^p(\cdot)} e_n^p(\cdot)}{\mathcal{E}x_m(\cdot)}\right)e_n^p(\cdot)\\ &= \frac{\sum_{n=1}^{n\in\mathcal{N}_m^p(\cdot)} e_n^p(\cdot)}{\mathcal{E}x_m(\cdot)}e_n^p(\cdot)\end{aligned} \tag{20}$$

Therefore, using Definitions 4 and 5, we redefine the utility function $\mathbb{U}_n^{\tilde{m}}(\cdot)$ as follows:

$$\mathbb{U}_n^{\tilde{m}}(\cdot) = E_n^{max}\tan^{-1}\left(e^{-\frac{\tilde{d}_n^p(\cdot)-d_n^p(\cdot)}{\tilde{d}_n^p(\cdot)}}\right) - p_{\tilde{m}}(\cdot)\tilde{d}_n^p(\cdot) \tag{21}$$

2) Utility function of a micro-grid: In the proposed scheme, ENTRANT, each micro-grid $m \in \mathcal{M}$ makes profit by selling the excess amount of energy to the set of connected PHEVs, i.e., $\mathcal{N}^p(\cdot)$. Each micro-grid m calculates the price coefficient, $p_m(\cdot)$, based on the amount of requested energy by the PHEVs $\mathcal{N}_m^p(\cdot)$, i.e., $\sum_{n=1}^{n\in\mathcal{N}_m^p(\cdot)}\tilde{d}_n^p(\cdot)$. The utility of each micro-grid $m \in \mathcal{M}$, i.e., $\mathbb{B}_m^p(\cdot)$, represents the profit of each micro-grid m by selling the excess amount of energy. Therefore, we define the utility function, $\mathbb{B}_m^p(\cdot)$, of each micro-grid $m \in \mathcal{M}$ as follows:

$$\mathbb{B}_m^p(\cdot) = [p_m(\cdot) - c_m(\cdot)]\sum_{n=1}^{n\in\mathcal{N}_m^p(\cdot)}\tilde{d}_n^p(\cdot) \tag{22}$$

where $c_m(\cdot)$ is the generation-cost coefficient of each micro-grid $m \in \mathcal{M}$.

In ENTRANT, each PHEV $n \in \mathcal{N}$ and each micro-grid $m \in \mathcal{M}$ try to maximize the payoff of the utility function, individually, following the proposed non-cooperative game theoretic approach.

B. Existence of Generalized Nash Equilibrium Solution

We determine the existence of generalized Nash equilibrium solution using *variational inequality* (VI) [31] as shown in Theorem 1.

Theorem 1. *Given a fixed amount of energy to be consumed by each PHEV, there exists a generalized Nash equilibrium solution, as there exists a variational inequality solution, for each PHEV n and each micro-grid m. Hence, each PHEV selects micro-grid $\tilde{m}$ over micro-grid m and each micro-grid decides the price coefficient, $p_m(\cdot)$, while satisfying the following constraints:*

$$\mathbb{U}_n^{\tilde{m}}(\cdot) \geq \mathbb{U}_n^m(\cdot), \qquad \textit{where } \forall m, \tilde{m} \in \mathcal{M} \tag{23}$$

$$\mathbb{B}_m^{p*}(\cdot) \geq \mathbb{B}_m^p(\cdot) \tag{24}$$

where $\mathbb{B}_m^{p*}$ *is the utility function of micro-grid* m *at Nash equilibrium point.*

Proof: The utility function of each PHEV n, i.e., $\mathbb{U}_n^m(\cdot)$, and the utility function of each micro-grid m, i.e., $\mathbb{B}_m^p(\cdot)$, need to be maximized. Hence, applying Karush-Kuhn-Tucker (KKT) conditions, we get:

$$\nabla_n \mathbb{U}_n^m(\cdot) = \nabla_n \lambda_n(\cdot) \left[\mathcal{E}x_m(\cdot) - \sum_{n=1}^{n \in \mathcal{N}_m^p(\cdot)} \tilde{d}_n^p(\cdot) \right],$$
$$\nabla_n \lambda_n(\cdot) \left[\mathcal{E}x_m(\cdot) - \sum_{n=1}^{n \in \mathcal{N}_m^p(\cdot)} \tilde{d}_n^p(\cdot) \right] = 0, \text{ and } \lambda_n(\cdot) > 0 \tag{25}$$

where $\lambda_n(\cdot)$ is the Lagrangian constant. Considering an overall utility function, we get:

$$\boldsymbol{\nabla} \boldsymbol{\mathcal{U}}^m(\cdot) - \boldsymbol{\nabla} \boldsymbol{\lambda}(\cdot) [\mathcal{E}x_m(\cdot) - \sum_{n=1}^{n \in \mathcal{N}_m^p(\cdot)} \tilde{d}_n^p(\cdot)] = 0 \tag{26}$$

where $\boldsymbol{\mathcal{U}}^m(\cdot) = \sum \mathbb{U}_n^m(\cdot)$, and $\boldsymbol{\lambda} \triangleq \lambda_1 \triangleq \cdots \triangleq \lambda_{|\mathcal{N}_m^p(\cdot)|}$. Hence, We get the Jacobian matrix of $\boldsymbol{\mathcal{U}}^m(\cdot)$ as follows:

$$\boldsymbol{J} \boldsymbol{\mathcal{U}}^m(\cdot) = \begin{bmatrix} K_1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & K_n & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & K_{|\mathcal{N}_m^p(\cdot)|} \end{bmatrix} \tag{27}$$

where $K_n = \frac{E_n^{max} e_n^p}{[\tilde{d}_n^p]^2 + [\bar{d}_n^p]^2} e^{-\left(\frac{\tilde{d}_n^p}{\bar{d}_n^p}\right)} - p_m(\cdot)$.

$\boldsymbol{J} \boldsymbol{\mathcal{U}}^m(\cdot)$ is a positive diagonal matrix, as we assume that the amount of requested energy for each PHEV n is non-negative. Therefore, we conclude that there exists variational inequality solution, i.e., generalized Nash equilibrium solution. ■

C. Algorithms

In order to reach the equilibrium of energy trading networks using the proposed scheme, ENTRANT, the PHEVs and the micro-grids take their respective strategies, while incurring marginal communication overhead. In this paper, we propose two different algorithms — (a) for PHEV, and (b) for micro-grid, as shown in Algorithms 1 and 2, respectively. Each PHEV n decides the amount of energy to be requested to the selected micro-grid using Algorithm

1. On the other hand, each micro-grid m calculates the price coefficient based on the amount of energy requested by the connected PHEVs using Algorithm 2, and broadcasts the calculated price coefficient, $p_m(\cdot)$.

Algorithm 1: ENTRANT Algorithm for PHEV

Inputs : E_n^{max}: Maximum battery capacity of PHEV n
$E_n^{res}(\cdot)$: Residual energy of PHEV n
$d_n^p(\cdot)$: Current value of request energy by PHEV n
$p_m(\cdot)$: price co-efficient of each micro-grid $m \in \mathcal{M}$
Outputs: $\tilde{d}_n^p(\cdot)$: Modified value of request energy by PHEV n
$\tilde{m}$: Selected micro-grid for energy supply

Calculate $\tilde{d}_n^p(\cdot)$ using following equation:
$\tilde{d}_n^p(\cdot) = (1 + \frac{E_n^{res}(\cdot)}{E_n^{max}})d_n^p(\cdot)$
// $\frac{E_n^{res}(\cdot)}{E_n^{max}}$ `is allowable change in energy request`
Calculate $\mathbb{U}_n^m(\cdot)$, where $\forall m \in \mathcal{M}$
if $\max \mathbb{U}_n^m(\tau) > \mathbb{U}_n^{\tilde{m}}(\tau - 1)$ **then**
    if $\mathbb{U}_n^{\tilde{m}}(\cdot) \geq \mathbb{U}_n^m(\cdot)$ **then**
        Request micro-grid $\tilde{m}$ to supply $\tilde{d}_n^p(\cdot)$ amount of energy
    else
        Request micro-grid m to supply $\tilde{d}_n^p(\cdot)$ amount of energy
    end
else
    $\tilde{d}_n^p(\cdot) = d_n^p(\cdot)$
    // `Nash Equilibrium reached`
    Request previously selected micro-grid $\tilde{m}$ to supply $\tilde{d}_n^p(\cdot)$ amount of energy
    // `Here,` $\tilde{m}$ `is the selected micro-grid in the previous iteration`
end
return

Algorithm 2: ENTRANT Algorithm for micro-grid

Inputs : $\tilde{d}_n^p(\cdot)$: Amount of request energy by PHEV n
$c_m(\cdot)$: Generation-cost coefficient of micro-grid m
$\mathcal{E}x_m(\cdot)$: Excess amount of generated energy
Output: $p_m(\cdot)$: Price coefficient of micro-grid m

Calculate $p_m(\cdot)$ using Equation (7)
if $\mathbb{B}_m^p(\tau) == \mathbb{B}_m^p(\tau - 1)$ **then**
    // `Nash Equilibrium reached`
end
Broadcast the price coefficient $p_m(\cdot)$
return

V. Performance Evaluation

A. Simulation Parameters

For performance evaluation, we consider randomly generated positions of the micro-grids, and the initial positions of the PHEVs on a MATLAB simulation platform, as shown in Table I. In this work, we assumed that each PHEV follows the Gauss-Markov mobility model. Therefore, we calculated the position of the PHEVs using Equations (9), (11), and (12). We considered randomly generated values for maximum battery capacity of the PHEVs.

TABLE I: Simulation Parameters

Parameter	Value
Simulation area	20 km×20 km
Number of micro-grids	4
Number of PHEVs	500
Maximum battery capacity	35-65 MWh
Residual energy of each PHEV	>10 MWh
Excess energy per micro-grid	99 MWh

In a coalition, each residential customer, i.e., home-users, decides his/her energy consumption profile *a priori*. Hence, based on the amount of energy generated by the micro-grids using renewable energy resources, each micro-grid calculates the amount of excess energy generated. For the sake of simulation, we considered that each micro-grid has a fixed amount of excess energy, i.e., 90 MWh [32], and the residual energy at the PHEV-end is generated randomly. Hence, based on the amount of requested energy by the connected PHEVs, each micro-grid decides the price coefficient, and the price to be paid by each customer.

B. Benchmark

The performance of the proposed scheme, energy trading network topology control (ENTRANT), is evaluated by comparing the results with other energy trading policies, such as the economics of electric vehicle charging (E2VC) [32], the energy trading without any game-theoretic approach (WoENT).

We refer to these different energy trading policies as ENTRANT, E2VC, and WoENT, through the rest of the paper. In E2VC [32], the authors proposed a non-cooperative game theoretic approach. Though the authors did not consider the choice of multiple micro-grids for each PHEV available in the coalition. In WoENT, we considered that each PHEV chooses the appropriate micro-grid from the available micro-grids based on the minimum distance to be traveled. Thus, we can improve the satisfaction factor of the PHEVs, and the energy load to each micro-grid using the proposed scheme, ENTRANT, than using other approaches, i.e., E2VC and WoENT.

C. Performance Metrics

(i) *Consumed energy per iteration*: The utilization of excess amount of energy generated can be visualized with the amount of consumed energy per iteration. In each iteration, with the increase in consumed energy by the PHEVs, the satisfaction factor of the micro-grids increases, as higher amount of energy is consumed by the PHEVs.

(ii) *Energy price per micro-grid*: Each PHEV wants to consume energy with lower price. However, if the energy-load to any micro-grid becomes higher, the price per unit energy of that micro-grid becomes high, while using

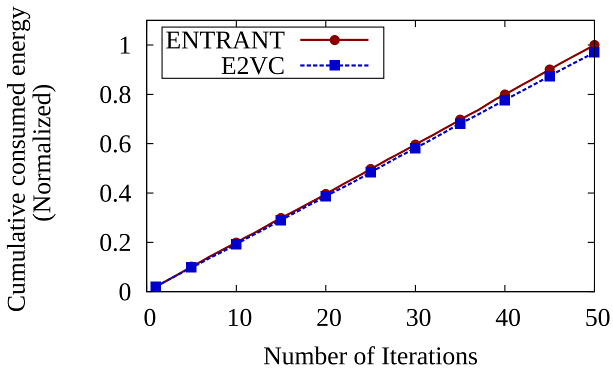

Fig. 3: Consumed Energy per iteration

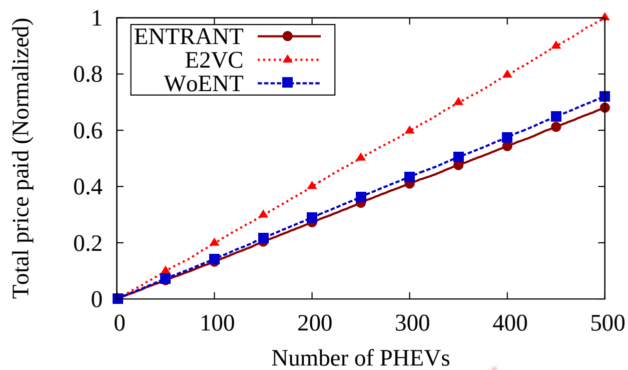

Fig. 4: Price paid by the PHEVs

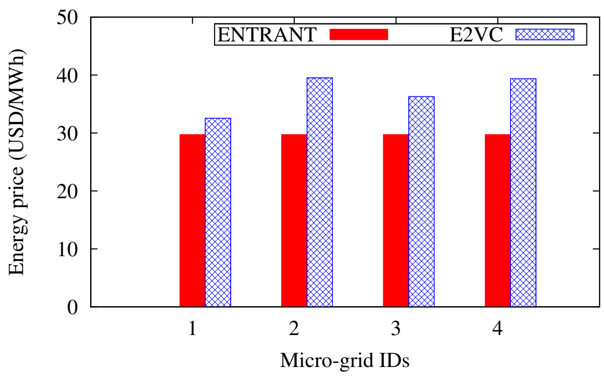

Fig. 5: Energy price per micro-grid

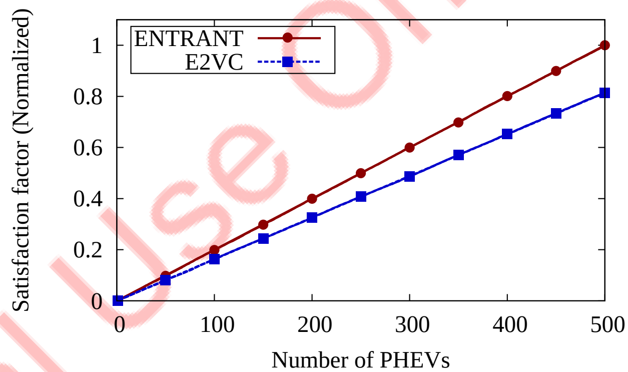

Fig. 6: Satisfaction factor of the PHEVs

dynamic pricing strategy. Hence, to utilize the excess amount of generated energy of each micro-grid, we need to distribute the energy request such that the price per unit energy becomes moderated, i.e., neither too high nor too low.

(iii) *Price paid per PHEV*: Based on the price per unit energy decided by the micro-grids and amount of energy to consumed by each PHEV, each micro-grid decides the amount of billing for each PHEV. However, the price decided by the micro-grids and the amount of energy requested by each PHEV are interdependent. If the amount of energy requested by each micro-grid becomes too high, the price becomes high. As a result, each PHEV needs to re-decide the amount of energy to consumed and request the selected micro-grid. On the other hand, if the price per unit energy decided by the micro-grid becomes too low, each PHEV requests high amount of energy. Therefore, the price per unit energy becomes high.

(iv) *Satisfaction factor of PHEVs*: Satisfaction factor of each PHEV is defined as the ratio of the amount of energy consumed, and the total demand of a PHEV. Hence, higher satisfaction factor signifies higher portion of required energy is served by the micro-grid. Each PHEV behaving rationally tries to maximize its satisfaction factor by consuming higher amount of energy.

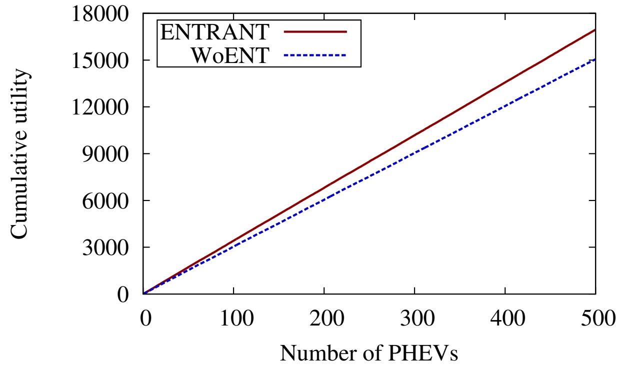

Fig. 7: Utility value of the PHEVs

(v) *Quality of energy service*: We consider that higher utility value signifies higher quality of energy service. Therefore, each PHEV tries to get higher quality of energy service by maximizing the payoff of the its utility function.

D. Results and Discussions

For simulation purpose, we assume that each micro-grid calculates the real-time supply and demand in every 10 seconds interval.

Figure 3 shows that the cumulative energy consumed per iteration is higher using the proposed scheme, ENTRANT, than using E2VC. Therefore, we conclude that within a coalition, each PHEV consumes higher amount of energy using ENTRANT, than using E2VC. Therefore, utilization of generated energy is much higher using ENTRANT, than using E2VC.

In Figure 4, the variation of the price paid by the PHEVs within a coalition is shown. Using ENTRANT, the PHEVs have to pay less, as the energy is properly distributed within the available micro-grids within a coalition. Hence, we conclude that in a coalition, the PHEVs gets the required energy by paying less while using the proposed scheme, ENTRANT, than using E2VC and WoENT.

Figure 5 shows that the price per unit energy, i.e., USD/MWh, is lower using ENTRANT, than using E2VC. Using ENTRANT, the price per unit energy is almost similar for each micro-grid, as the energy load is properly distributed within the available micro-grids within a coalition.

Figure 6 shows that the cumulative satisfaction factor is much higher using ENTRANT, than using E2VC. Therefore, we conclude that using the proposed scheme, ENTRANT, each PHEV consumes higher percentage of energy of its requirement to charge its battery fully.

Figure 7 shows that the payoff, i.e., utility, of the utility function of each PHEV is much higher using ENTRANT, than using WoENT. Hence, we conclude that each PHEV can get higher quality of energy service using ENTRANT, than using WoENT.

VI. Conclusion

In this paper, we formulated a multi-leader multi-follower Stackelberg game theoretic approach to study the problem of energy trading network topology control. Based on the proposed approach, i.e, ENTRANT, we showed how energy can be distributed within the PHEVs within a coalition having multiple micro-grids. The simulation results show that the proposed scheme, ENTRANT, yields improved results.

Future extension of this work includes understanding how the energy trading network topology can be controlled in advance based on the estimated trajectory of the PHEVs, so that scheduling in energy trading has less delay, and the the energy can be properly distributed within a single coalition, i.e., multiple micro-grids, or multiple coalitions.

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