Two-Dimensional Array

Declaration

A 2-D array within a function is declared as follows:

```
#define ROW 3
#define COL 5
..... what(....){
      int a[ROW][COL] .... ;
..........................
}
```

Logical View

Logically it may be viewed as a two-dimensional collection of data, three rows and five columns, each location is of type `int`.

Columns

	0	1	2	3	4
0	a[0][0]	a[0][1]	a[0][2]	a[0][3]	a[0][4]
1	a[1][0]	a[1][1]	a[1][2]	a[1][3]	a[1][4]
2	a[2][0]	a[2][1]	a[2][2]	a[2][3]	a[2][4]

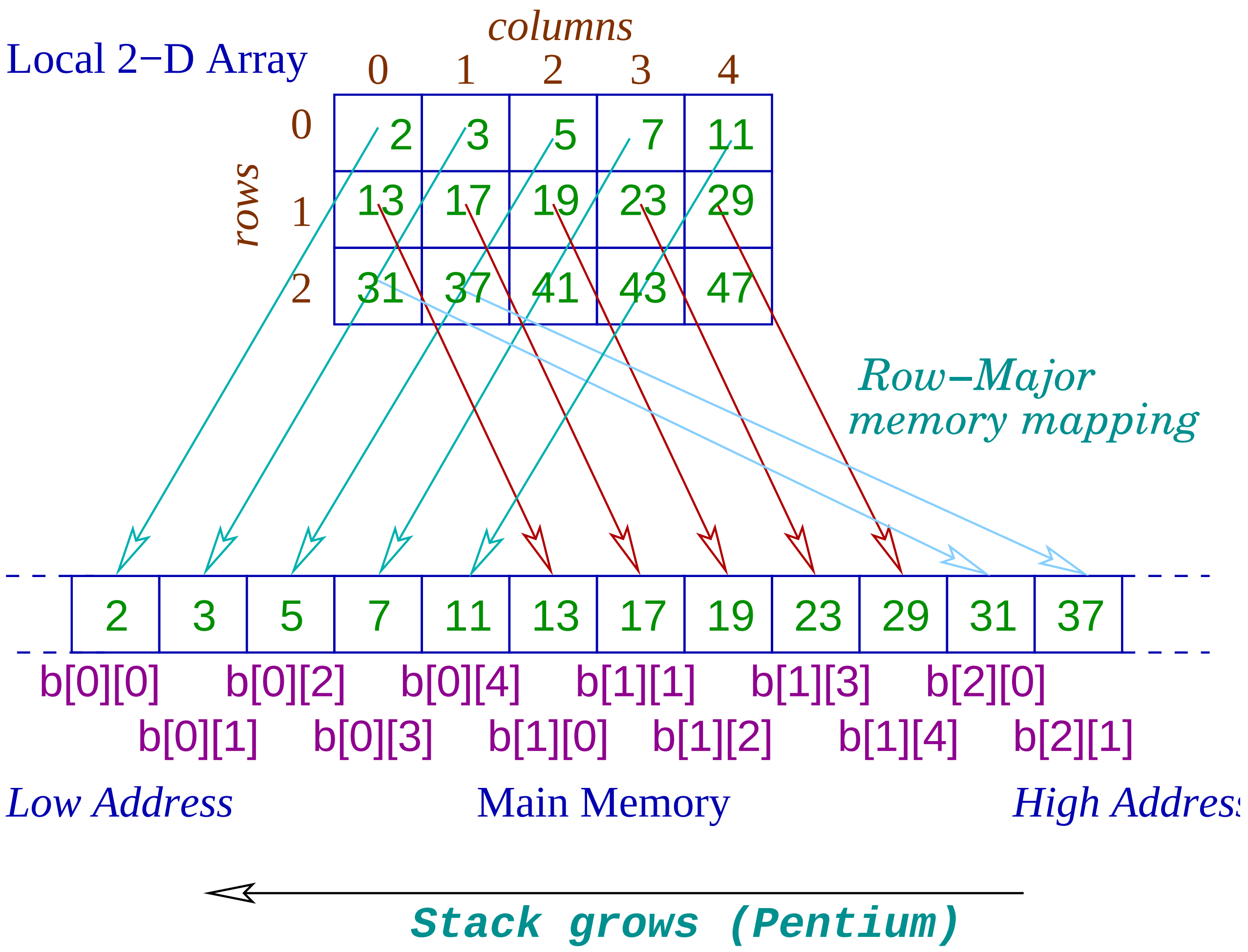
Memory Mapping

The computer memory is an one-dimensional sequence of bytes. C compiler stores the two-dimensional[a] object in row-major order in the memory[b].

---

[a]Multi-dimensional in general.

[b]It is stored in column-major order in some other programming languages e.g. FORTRAN.


Local 2−D Array
columns
0 1 2 3 4
rows
0 1 2
2 3 5 7 11
13 17 19 23 29
31 37 41 43 47
Row−Major
memory mapping
2 3 5 7 11 13 17 19 23 29 31 37
b[0][0] b[0][1] b[0][2] b[0][3] b[0][4] b[1][0] b[1][1] b[1][2] b[1][3] b[1][4] b[2][0] b[2][1]
Low Address
Main Memory
High Address
Stack grows (Pentium)

I/O

Data can be read in a 2-D array and data can be printed from a 2-D array, one element at time[a]. Consider the following 3×5 matrix of real numbers. We can read the matrix in a 2-D array and print it in a C program.

$$\begin{bmatrix} 1.0 & 2.0 & 3.0 & 4.0 & 5.0 \\ -1.0 & -2.0 & -3.0 & -4.0 & -5.0 \\ 10.0 & 20.0 & 30.0 & 40.0 & 50.0 \end{bmatrix}$$

[a] A string can be read as a whole.

```
#include <stdio.h>
#define MAXROW 50
#define MAXCOL 50
int main() // matRdWr.c
{
    double a[MAXROW][MAXCOL];
    int   rows, columns, i, j;

    printf("Enter the number of Rows: ") ;
    scanf("%d", &rows) ;
    printf("\nEnter the number of Columns: ") ;
    scanf("%d", &columns) ;
    printf("\nEnter row-wise, ");
    printf("the elements of the matrix\n") ;
```

```
    for(i = 0; i < rows; ++i)
      for(j = 0; j < columns; ++j)
         scanf("%lf", &a[i][j]) ;
    putchar('\n') ;
    printf("The matrix is:\n") ;
    for(i = 0; i < rows; ++i) {
      for(j = 0; j < columns; ++j)
         printf("%4.2f ", a[i][j]) ;
      putchar('\n') ;
    }
    return 0;
}
```

Data File

It is tedious to enter data manually. So we use a data file dataMat and redirect the input from the file.

```
3 5
1.0 2.0 3.0 4.0 5.0
-1.0 -2.0 -3.0 -4.0 -5.0
10.0 20.0 30.0 40.0 50.0
```

Running the Code

```
$ cc -Wall matRdWr.c
$ a.out < dataMat
Enter the number of Rows:
Enter the number of Columns:
Enter row-wise, the elements of the
matrix
The matrix is:
1.00 2.00 3.00 4.00 5.00
-1.00 -2.00 -3.00 -4.00 -5.00
10.00 20.00 30.00 40.00 50.00
```

Initialization of 2-D Array

```
#include <stdio.h>
#define MAXROW 5
#define MAXCOL 5
int main() // init2D.c
{
    int a[MAXROW][MAXCOL], i, j,
        b[MAXROW][MAXCOL] = {{0, 1, 2, 3, 4},
                             {10, 20, 30, 40, 50},
                             {15, 25, 35, 45, 55},
                             {50, 51, 52, 53, 54},
                             {55, 55, 55, 55, 55},
                            },
```

```
c[MAXROW][MAXCOL] = {{10, 20, 30},
                     {40, 50, 60, 70, 80},
                    },
d[][MAXCOL] = {{2, 4, 6, 8, 0},
               {4, 6, 8, 0, 2} } ,
e[MAXROW][MAXCOL] = {0, 1, 2, 3, 4,
                     5, 6, 7, 8, 9,
                     10, 11, 12, 13, 14,
                     15, 16, 17, 18, 19,
                     20, 21, 22, 23, 24
                    },
f[][MAXCOL] = {2, 4, 6, 8, 0,
               4, 6, 8, 0, 2
              } // ,
```

```
//          g[MAXROW][] = {{0, 1, 2, 3, 4},
//                         {10, 20, 30, 40, 50},
//                         {15, 25, 35, 45, 55},
//                         {50, 51, 52, 53, 54},
//                         {55, 55, 55, 55, 55},
//                        }
                          ;
    printf("\n") ;
    printf("Array a[][]\n") ;

    for(i = 0; i < MAXROW; ++i) {
      for(j = 0; j < MAXCOL; ++j)
         printf("%d ", a[i][j]) ;
      printf("\n") ;
```

```
}
printf("\n") ;
printf("Array b[][]\n") ;
for(i = 0; i < MAXROW; ++i) {
  for(j = 0; j < MAXCOL; ++j)
     printf("%d ", b[i][j]) ;
  printf("\n") ;
}

printf("\n") ;
printf("Array c[][]\n") ;
for(i = 0; i < MAXROW; ++i) {
  for(j = 0; j < MAXCOL; ++j)
     printf("%d ", c[i][j]) ;
```

```
      printf("\n") ;
    }

    printf("\n") ;
    printf("Array d[][]\n") ;
    for(i = 0; i < MAXROW; ++i) {
      for(j = 0; j < MAXCOL; ++j)
          printf("%d ", d[i][j]) ;
      printf("\n") ;
    }

    printf("\n") ;
    printf("Array e[][]\n") ;
    for(i = 0; i < MAXROW; ++i) {
```

```
      for(j = 0; j < MAXCOL; ++j)
         printf("%d ", e[i][j]) ;
      printf("\n") ;
    }

    printf("\n") ;
    printf("Array f[][]\n") ;
    for(i = 0; i < MAXROW; ++i) {
      for(j = 0; j < MAXCOL; ++j)
         printf("%d ", f[i][j]) ;
      printf("\n") ;
    }
    return 0;
}
```

What is ‘b’?

```
int a[10], b[5][3];
```

We know that ‘a’ is a constant expression whose value is the address of the 0^{th} location of the array `a[10]`. Similarly `a + i` is the address of the i^{th} location of the array.

What is ‘b’ and what is its arithmetic?

Arithmetic of `b[5][3]`

Consider the following program:

```
#include <stdio.h>
int main() //  2DArith1.c
{
    int a[10], b[3][5];

    printf("a: %p\tb = %p\n", a, b) ;
    printf("a+1: %p\tb+1: %p\n", a+1,b+1) ;
    printf("a+2: %p\tb+2: %p\n", a+2,b+2) ;
    printf("a+3: %p\tb+3: %p\n", a+3,b+3) ;
    return 0;
}
```

Output

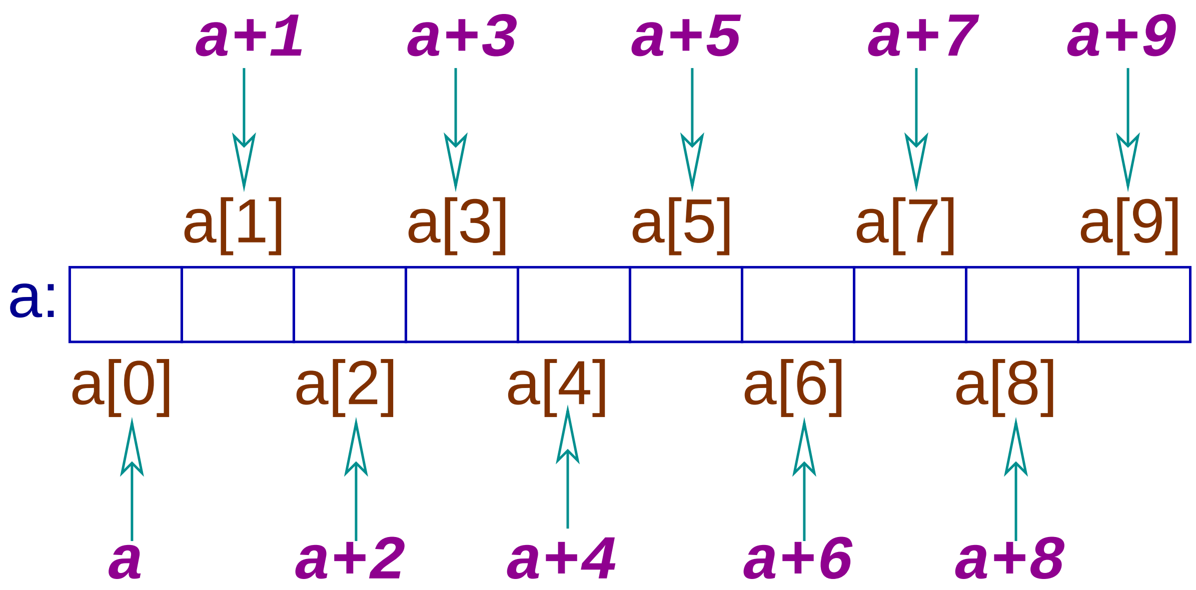
```
$ cc -Wall 2DArith1.c
$ a.out
a:  0xbfec6a90 b = 0xbfec6a50
a+1:  0xbfec6a94 b+1:  0xbfec6a64
a+2:  0xbfec6a98 b+2:  0xbfec6a78
a+3:  0xbfec6a9c b+3:  0xbfec6a8c
```

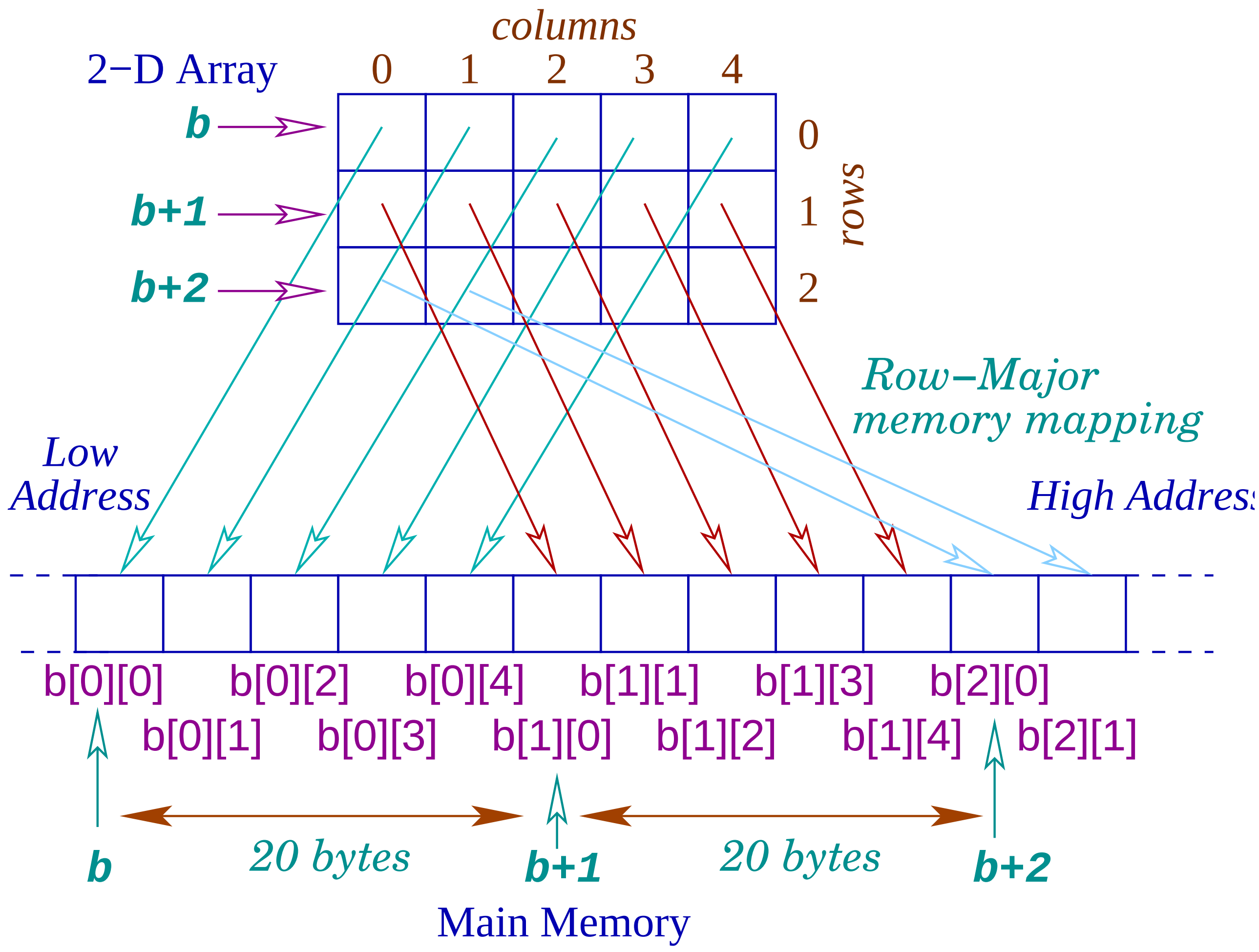
Increment of ‘`a`’ is by 4-bytes, `sizeof(int)`, but the increment of ‘`b`’ is by 20-bytes. The question is why?

Row-Major Space Allocation

The answer lies in the row-major memory space allocation of 2-D array by the C compiler.

1–D Array





columns
2−D Array
0 1 2 3 4
b
b+1
b+2
0
1
2
rows
Row−Major
memory mapping
Low
Address
High Address
b[0][0]
b[0][2]
b[0][4]
b[1][1]
b[1][3]
b[2][0]
b[0][1]
b[0][3]
b[1][0]
b[1][2]
b[1][4]
b[2][1]
b
20 bytes
b+1
20 bytes
b+2
Main Memory

Arithmetic of ‘`b`’

- `b` is the address of the 0^{th} row.
- `b+1` is the address of the 1^{st} row.
- `b+i` is the address of the i^{th} row.

The size of a row is

$$\begin{aligned} & c \times \texttt{sizeof(int)} \\ = & \; 5 \times \texttt{sizeof(int)} \\ = & \; 5 \times 4 = 20 \text{ bytes} \end{aligned}$$

where c is the number of columns.

Arithmetic of ‘`b`’

The difference between $b + 1$ and b is 20 and that of `b+i` and `b` is $20i$.

`b+i` points to the i^{th} row

Type of ‘`b`’

‘`b`’ is a pointer constant of type `int [][5]`, a row of five `int`. If such a pointer is incremented, it goes up by $5\times$ `sizeof(int)` (number of bytes).
Type `int [][5]` is equivalent to `int (*)[5]`.

Arithmetic of `*(b+i)`

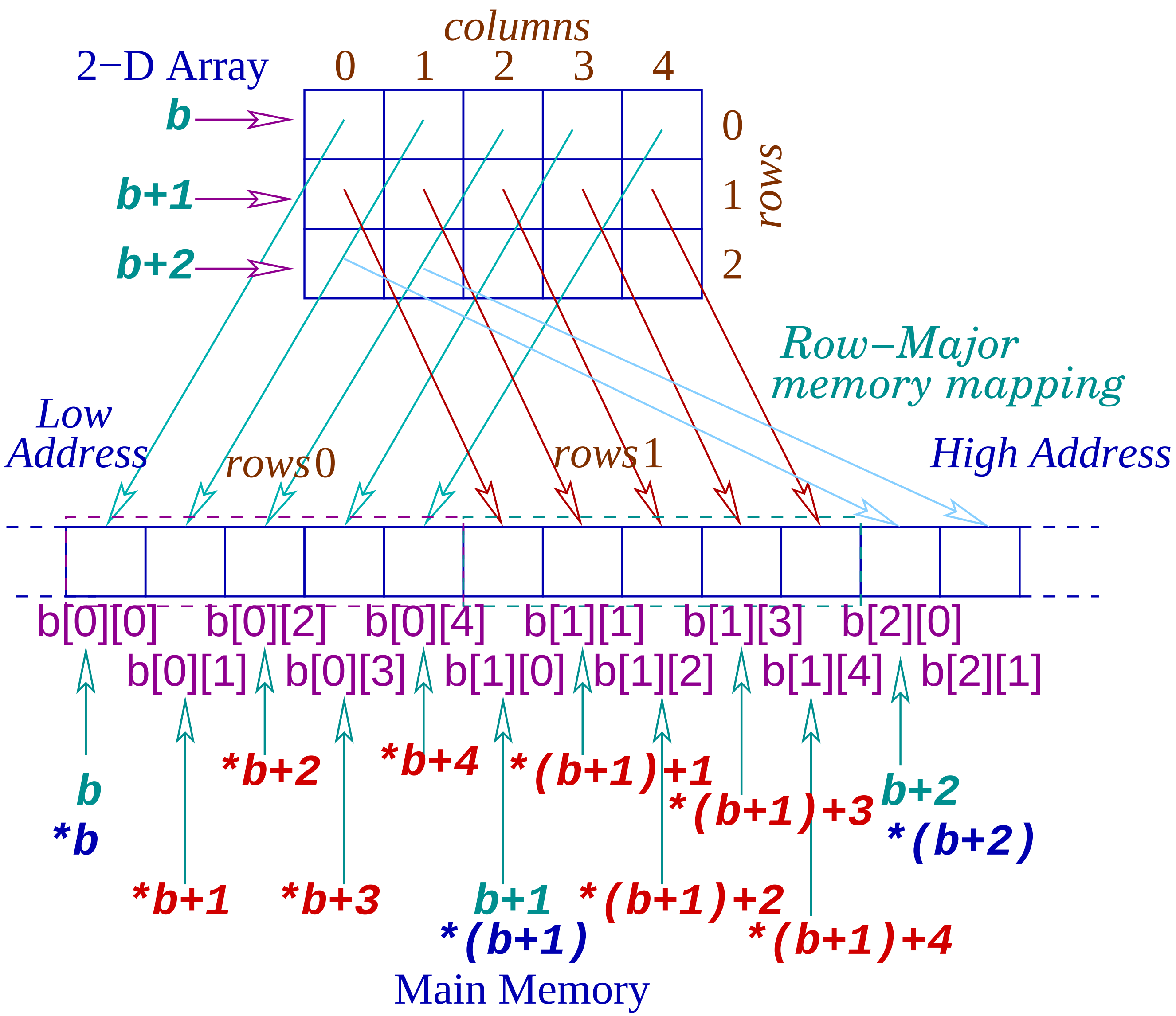
- If `b` is the address of the 0^{th} row, `*b` is the 0^{th} row itself. A row may be viewed as an 1-D array, so `*b` is the address of the 0^{th} element of the 0^{th} row.
- Similarly `b+i` is the address of the i^{th} row, `*(b+i)` is the i^{th} row, so `*(b+i)` is the address of the 0^{th} element of the i^{th} row.

Arithmetic of `*(b+i)`

- If `*b` is the address of the 0^{th} element of the 0^{th} row, `*b + 1` is the address of the 1^{st} element of the 0^{th} row.
- Similarly `*b + j` is the address of the j^{th} element of the 0^{th} row.
- The difference between `b + 1` and `b` is 20 (bytes) but the difference between `*b + 1` and *b is the sizeof(int), 4 (bytes).

Arithmetic of `*(b+i)`

- If `*(b+i)` is the address of the 0^{th} element of the i^{th} row, `*(b+i) + 1` is the address of the 1^{st} element of the i^{th} row.
- Similarly `*(b+i) + j` is the address of the j^{th} element of the i^{th} row.
- The difference between `b + i` and `b` is $20i$ (bytes), but the difference between `*(b + i) + j` and `*(b+i)` is $4j$ (bytes).



2−D Array
columns
0 1 2 3 4
b
b+1
b+2
0
1
2
rows
Row−Major
memory mapping
Low
Address
rows 0
rows 1
High Address
b[0][0]
b[0][1]
b[0][2]
b[0][3]
b[0][4]
b[1][0]
b[1][1]
b[1][2]
b[1][3]
b[1][4]
b[2][0]
b[2][1]
b
*b
*b+1
*b+2
*b+3
*b+4
b+1
*(b+1)
*(b+1)+1
*(b+1)+2
*(b+1)+3
*(b+1)+4
b+2
*(b+2)
Main Memory

C Program

```
#include <stdio.h>
int main() // 2DArith2.c
{
    int b[3][5] ;
    printf("b: %p\t*b: %p\n", b, *b) ;
    printf("b+1: %p\t*b+1: %p\n", b+1, *b+1) ;
    printf("b+2: %p\t*(b+2): %p\t*(b+2)+3: %p\n",
                b+2, *(b+2), *(b+2)+3) ;
    return 0;
}
```

Output

```
$ cc -Wall 2DArith2.c
$ a.out
b:  0xbfeb3360 *b:  0xbfeb3360
b+1:  0xbfeb3374 *b+1:  0xbfeb3364
b+2:  0xbfeb3388 *(b+2):  0xbfeb3388
*(b+2)+3:  0xbfeb3394
```

```
*(b + i) + j
```

We know that

- `b` is the address of the 0^{th} row,
- `b+i` is the address of the i^{th} row,
- `*(b+i)` is the address of the 0^{th} element of the i^{th} row,
- `*(b+i)+j` is the address of the j^{th} element of the i^{th} row,

`*(b + i) + j` and `&b[i][j]`

We know that

- `*(b+i)+j` is the address of the j^{th} element of the i^{th} row,
- `b[i][j]` is the j^{th} element of the i^{th} row,
- `&b[i][j]` is the address of the j^{th} element of the i^{th} row, so

`*(b + i) + j` is equivalent to `&b[i][j]`

`*(*(b + i) + j)` and `b[i][j]`

We know that `*(b+i)+j` is the address of the j^{th} element of the i^{th} row, so

`*(*(b + i) + j)` is equivalent to `b[i][j]`

Equivalences

- `*(*(b + i) + j)` is equivalent to `b[i][j]`
- `*(b + i) + j` is equivalent to `&b[i][j]`
- `*(b[i] + j)` is equivalent to `b[i][j]`
- `b[i] + j` is equivalent to `&b[i][j]`
- `(*(b+i))[j]` is equivalent to `b[i][j]`

We shall use the right-hand side notations

C Program

```
#include <stdio.h>
int main() // 2DArith3.c
{
    int b[3][5] = {{0,1,2,3,4},
                   {5,6,7,8,9},
                   {10,11,12,13,14}
                  };

    printf("b[2][3]: %d\n", b[2][3]);
    printf("(*(b+2))[3]: %d\n", (*(b+2))[3]);
    printf("*(b[2]+3): %d\n", *(b[2]+3));
    printf("*(*(b+2)+3): %d\n", *(*(b+2)+3));
```

```
    printf("&b[2][3]: %p\n", &b[2][3]);
    printf("*(b+2)+3: %p\n", *(b+2)+3);
    printf("b[2]+3: %p\n", b[2]+3);
    return 0;
}
```

Output

```
$ cc -Wall 2DArith3.c
$ a.out
b[2][3]:  13
(*(b+2))[3]:  13
*(b[2]+3):  13
*(*(b+2)+3):  13
&b[2][3]:  0xbfe94c44
*(b+2)+3:  0xbfe94c44
b[2]+3:  0xbfe94c44
```

Calculation of the Address of `b[i][j]`

Given the declaration `int b[3][5]`, the C compiler can calculate the address of the j^{th} element of the i^{th} row by the following formula:

$$b + k(5i + j)$$

where $k =$ `sizeof(int)`. Other than the value of row and column indices the compiler needs the starting address `b`, the number of columns 5 and the size of the data type.

C Program

```
#include <stdio.h>
#define COL 5
int main() // 2DArith4.c
{
    int b[3][COL], i=1, j=2;
    printf("&b[%d][%d]=%p\n",i,j,&b[i][j]);
    printf("(int*)(b+%d)+%d=%p\n",i,j,(int*)(b+i)+j);
    printf("(int)b+%d*(%d*%d+%d)=0x%x\n",sizeof(int),
              COL, i,j,(int)b+sizeof(int)*(COL*i+j));
    return 0;
}
```

Output

```
$ cc -Wall 2DArith4.c
$ a.out
&b[1][2]=0xbff6104c
(int *)(b+1)+2=0xbff6104c
(int)b+4*(5*1+2)=0xbff6104c
```

1-D Array and Formal Parameter

Consider the declaration `int a[10]`,

- the array name is a pointer constant.
- the formal parameter: `int x[]` or `int *x` is a pointer variable of the corresponding type, where the address of an array location is copied.
- These two information are sufficient for the compiler to compute the address of `x[i]`.

Formal Parameter for 2-D Array

Consider the declaration *type* `b[ROW][COL]`. C compiler needs the starting address `b`, the data type *type*, and the number of columns `COL` inside a called function to calculate the address of `x[i][j]` (values of `i` and `j` are information local to the function).

Formal Parameter for 2-D Array

The formal parameter looks like

```
...   but(type x[][COL] ...)
```

where `x` is a variable of type *type* `[][COL]`.

Matrix Multiplication

Consider the real matrices $[a_{ij}]_{p\times q}$ and $[b_{ij}]_{q\times r}$.
The product matrix $[c_{ij}]_{p\times r} = [a_{ij}]_{p\times q} \times [b_{ij}]_{q\times r}$,
where $c_{ij} = \sum_{k=1}^{q} a_{ik} \times b_{kj}$, for all i, $1 \leq i \leq p$
and all j, $1 \leq j \leq r$.
We can store the matrices in 2-D array and multiply.

C Code

```
#include <stdio.h>
#define MAXROW 50
#define MAXCOL 50
void matMult( // matMult.c
      double matA[][MAXCOL],
      double matB[][MAXCOL],
      double matC[][MAXCOL],
      int rowA, int colA, int colB
            ) {
     int i, j, k ;

     for(i = 0; i < rowA; ++i)
```

```
        for(j = 0; j < colB; ++j) {
            matC[i][j] = 0.0 ;
            for(k = 0; k < colA; ++k)
                matC[i][j] += matA[i][k]*matB[k][j] ;
        }
}
void readMatrix(
      char *name,
      double x[][MAXCOL],
      int row, int col
              ) {
     int i, j ;

     printf("Enter the matrix %s:\n", name) ;
```

```
    printf("Row-by-row\n") ;
    for(i = 0; i < row; ++i)
        for(j = 0; j < col; ++j)
            scanf("%lf", &x[i][j]);
}
void writeMatrix(
     char *name,
     double x[][MAXCOL],
     int row, int col
              ) {
    int i, j ;
    printf("The matrix %s:\n", name) ;
    for(i = 0; i < row; ++i) {
        for(j = 0; j < col; ++j)
```

```
                printf("%6.2f ", x[i][j]);
            printf("\n") ;
        }
}
int main() // matMult.c data matData
{
    double aMat[MAXROW][MAXCOL],
           bMat[MAXROW][MAXCOL],
           cMat[MAXROW][MAXCOL];
    int aRow, aCol, bCol;

    printf("Enter the row and column numbers of A\n");
    scanf("%d%d", &aRow, &aCol);
    readMatrix("A", aMat, aRow, aCol);
```

```
    printf("Enter the column numbers of B\n");
    scanf("%d", &bCol);
    readMatrix("B", bMat, aCol, bCol);
    writeMatrix("A", aMat, aRow, aCol);
    writeMatrix("B", bMat, aCol, bCol);
    matMult(aMat, bMat, cMat, aRow, aCol, bCol);
    writeMatrix("C", cMat, aRow, bCol);
    return 0;
}
```

Output

```
$ cc -Wall matMult.c
$ a.out < matData
Enter the row and column numbers of A
Enter the matrix A:
Row-by-row
Enter the column numbers of B
Enter the matrix B:
Row-by-row
The matrix A:
1.00 2.00 3.00 4.00
5.00 6.00 7.00 8.00
The matrix B:
0.00 2.00 3.00
4.00 0.00 6.00
```

```
7.00 8.00 0.00
1.00 5.00 6.00
The matrix C:
33.00 46.00 39.00
81.00 106.00 99.00
```

Type of `x` in `readMatrix()`

Consider the prototype

```
.... readMatrix(..., int x[][50], ..)
```

- `x` is single a variable of type pointer to an `int` i array of 50-locations,
- we can equivalently write
  `.. readMatrix(.., int (*x)[50], ..)`.
  Increment of `x` is a jump by $50\times$ `sizeof(int)` bytes.
- The parenthesis is essential, otherwise in
  `.... readMatrix(..., int *x[50], ..)`, `x` is a pointer to an `int` pointer

C Program

```
#include <stdio.h>
#define MAXROW 10
#define MAXCOL 50
void what(int x[][MAXCOL],int (*y)[MAXCOL]){
    printf("x: %u\tx+1: %u\n",
            (unsigned)x, (unsigned)(x+1)) ;
    printf("y: %u\ty+1: %u\n",
            (unsigned)y, (unsigned)(y+1)) ;
}
int main() // 2DArith5.c
{
    int a[MAXROW][MAXCOL] ;
```

```
    printf("a: %u\ta+1: %u\n",
        (unsigned)a, (unsigned)(a+1)) ;
    what(a,a) ;
    return 0;
}
```

Output

```
$ cc -Wall 2DArith5.c
$ a.out
a:  3220066416 a+1:  3220066616
x:  3220066416 x+1:  3220066616
y:  3220066416 y+1:  3220066616
```