Sorting

CS10003 PROGRAMMING AND DATA STRUCTURES

The Basic Problem

Given an array: `x[0], x[1], ... , x[size-1]` reorder the elements so that

`x[0] <= x[1] <= ...  <= x[size-1]`

- That is, reorder entries so that the list is in increasing (non-decreasing) order.

We can also sort a list of elements in decreasing (non-increasing) order.

We prefer not to use additional arrays for the element rearrangement.

Example

Original list:

```
10, 30, 20, 80, 70, 10, 60, 40, 70
```

Sorted in non-decreasing order:

```
10, 10, 20, 30, 40, 60, 70, 70, 80
```

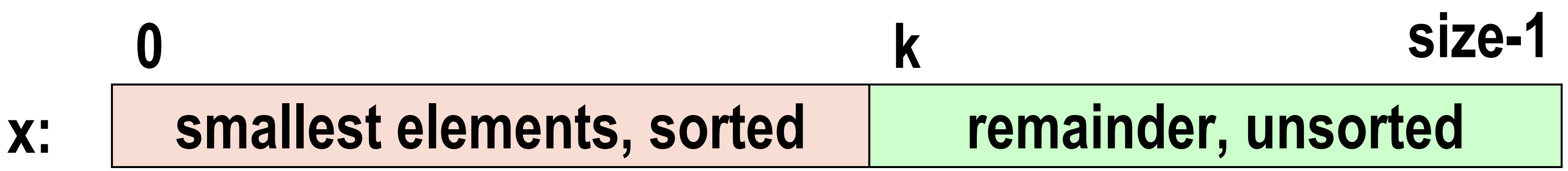
Sorted in non-increasing order:

```
80, 70, 70, 60, 40, 30, 20, 10, 10
```

Selection Sort

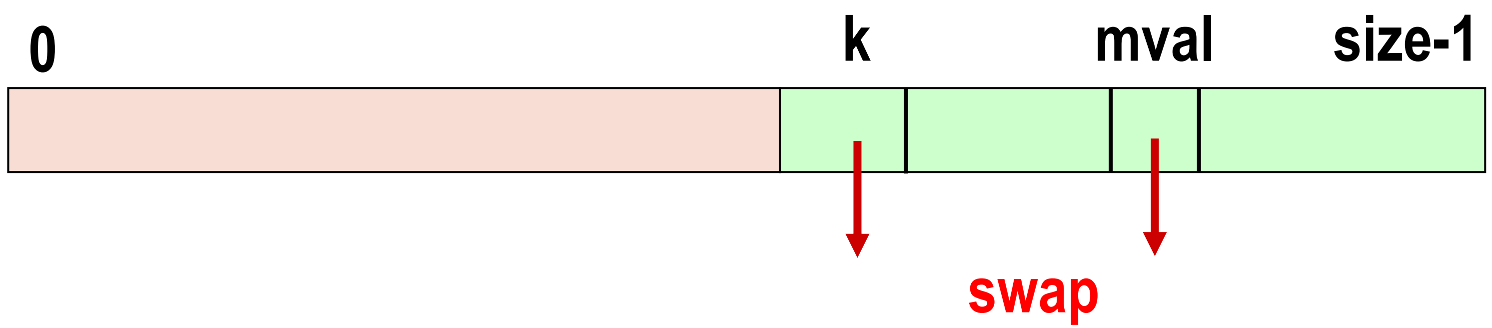
SELECTION SORT: The idea

General situation :



Steps:

- Initialize `k = 0`.
- Find smallest element, `mval`, in the array segment `x[k...size-1]`
- Swap smallest element with `x[k]`, then increase `k`.



Subproblem

```
/*  Find index of smallest element in x[k...size-1] */

int min_loc (int x[ ], int k, int size)
{
      int j, pos;

      pos = k;
      for (j=k+1; j<size; j++)
        if (x[j] < x[pos])
            pos = j;
      return pos;
}
```

Selection Sort Function

```
/* Sort x[0..size-1] in non-decreasing order */

int sel_sort (int x[], int size) {
        int k, m, temp;

        for (k = 0; k < size-1; k++) {
            m = min_loc (x, k, size);
                /* Swap x[k], x[m]*/
            temp = x[k];
            x[k] = x[m];
            x[m] = temp;
        }
}
```

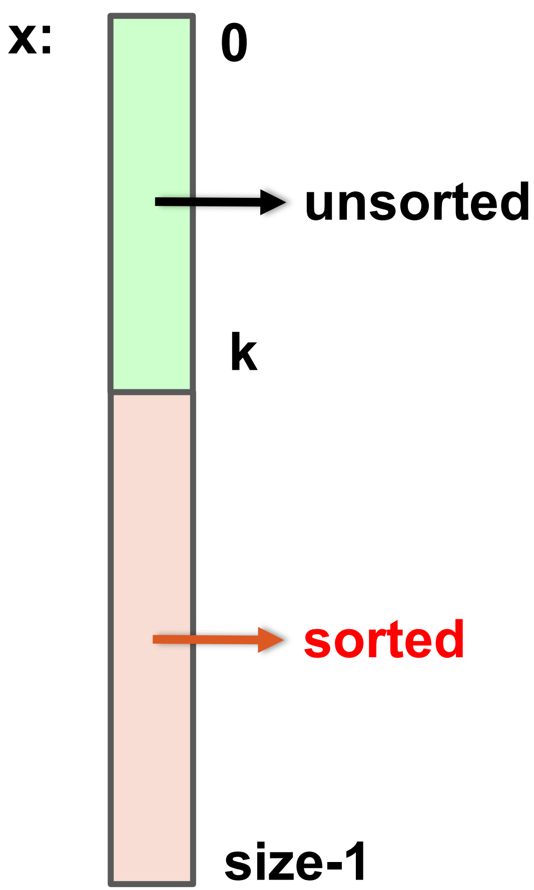
Example

x:	3	12	-5	6	142	21	-17	45
x:	-17	12	-5	6	142	21	3	45
x:	-17	-5	12	6	142	21	3	45
x:	-17	-5	3	6	142	21	12	45
x:	-17	-5	3	6	142	21	12	45
x:	-17	-5	3	6	12	21	142	45
x:	-17	-5	3	6	12	21	142	45
x:	-17	-5	3	6	12	21	45	142
x:	-17	-5	3	6	12	21	45	142

Bubble Sort

BUBBLE SORT: The idea

General situation:



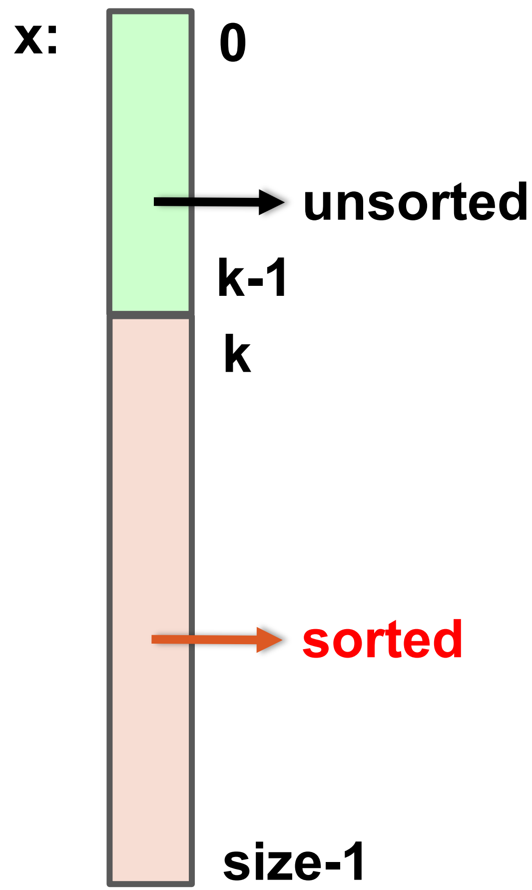
In every pass, we go on comparing neighboring pairs, and swap them if out of order.

```
for j = 0 to k-1
  if (x[j] > x[j+1])
      interchange them.
```

At the end of this iteration, the 'next largest' element (among the unsorted part) will settle at x[k].

Lighter elements bubble up.

Heavier elements settle down.



Bubble Sort

```
void bubble_sort (int x[], int size) {
    int t;
    for (i = 0; i < size; i++)
        for (j = 0; j < size-i-1; j++)
            if (x[j] > x[j+1]) {
             // swap a[j] and a[j+1]
                t = a[j];
                a[j] = a[j+1];
                a[j+1] = t;
            }
}
```

How do the passes proceed?

In pass 1, we consider index 0 to size-1
In pass 2, we consider index 0 to size-2
In pass 3, we consider index 0 to size-3
……
……
In pass size-1, we consider index 0 to 1.

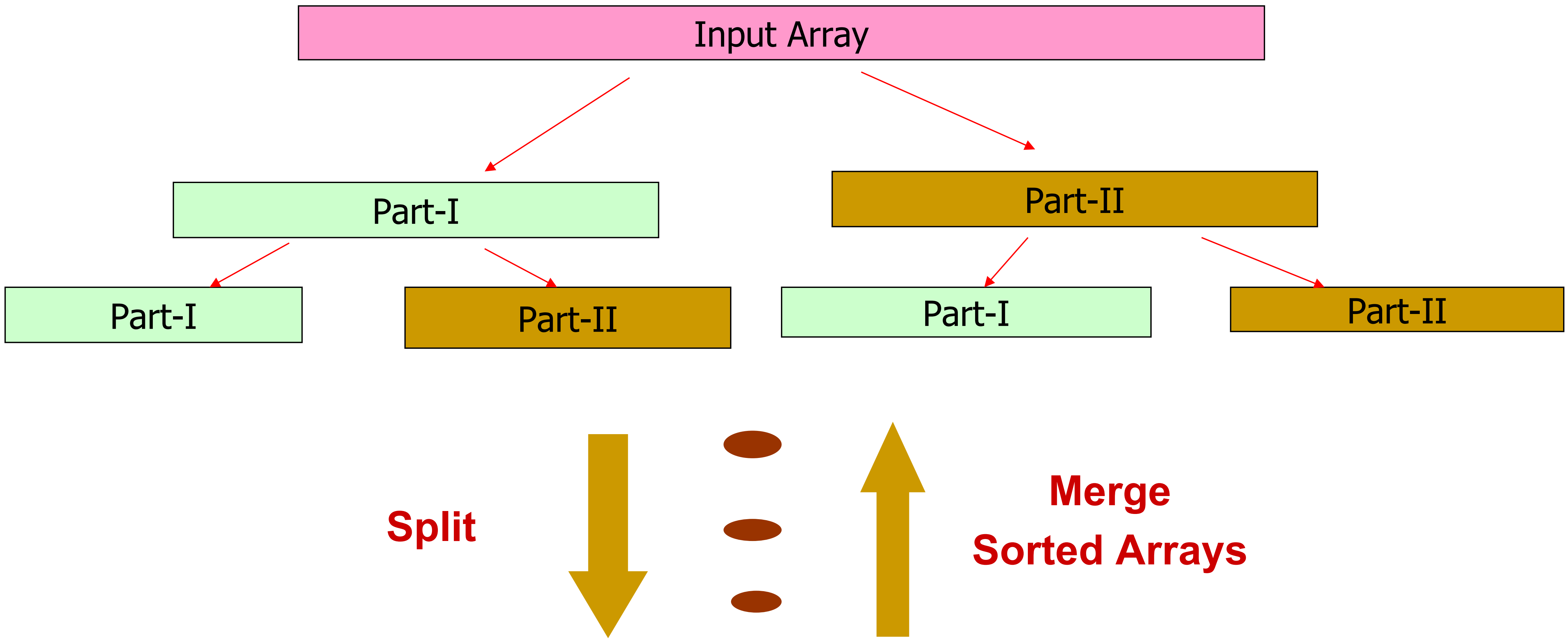
A more efficient sorting method: Mergesort

A popular sorting algorithm based on the divide-and-conquer approach.

Basic idea (divide-and-conquer method)

```
sort (list)
{
    if the list has length greater than 1
    {
            Partition the list into lowlist and highlist;
            sort (lowlist);
            sort (highlist);
            combine (lowlist, highlist);
    }
}
```

Merge Sort

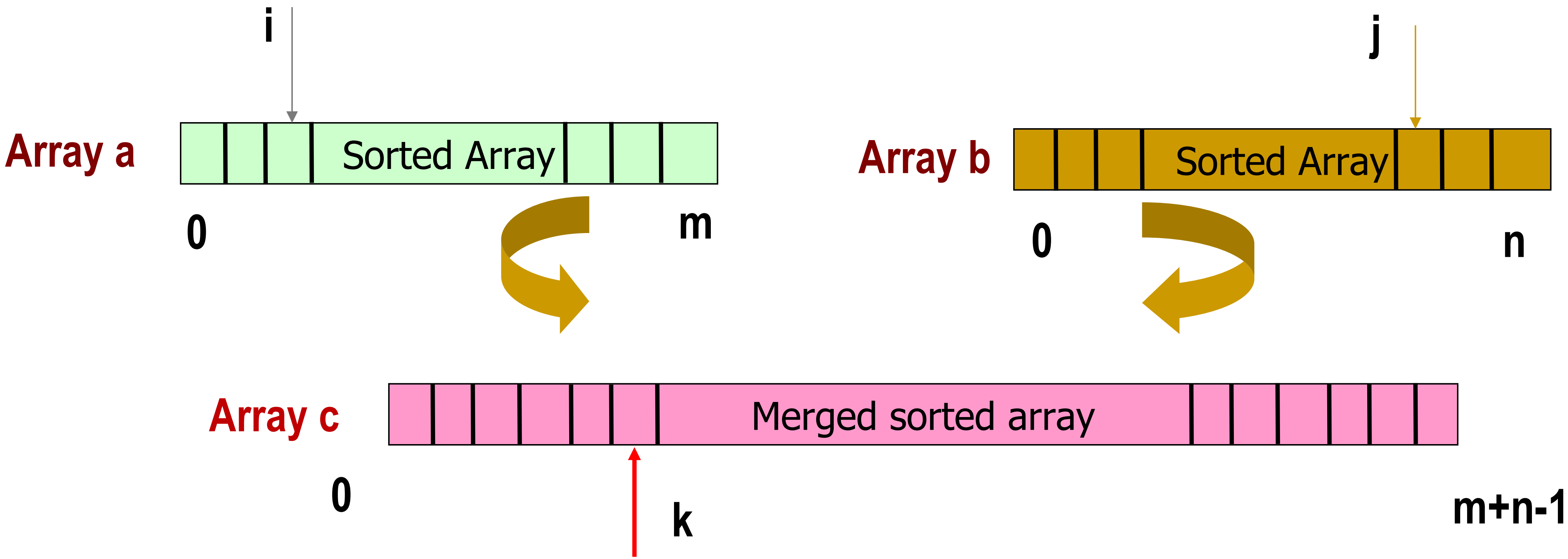


Input Array
Part-I
Part-II
Part-I
Part-II
Part-I
Part-II
Split
Merge
Sorted Arrays

```
void merge_sort (int *A, int n)
{
    int i, j, k, m;
    int *B, *C;

    if (n > 1)  {
       k = n/2;   m = n - k;
       B = (int *) malloc (k * sizeof(int));
       C = (int *) malloc (m * sizeof(int));
       for (i=0; i<k; i++)  B[i] = A[i];
              for (j=k; j<n; j++)  C[j-k] = A[j];
              // B contains first half of A
              // C contains second half of A
       merge_sort (B, k);
       merge_sort (C, m);
       merge (B, C, A, k, m); // destination array is A
       free(B); free(C);
    }
}
```

Merging two sorted arrays



Copy element from a (indexed by i) if its value is smaller than the element in b pointed by j ; otherwise, copy the element from b (indexed by j).

If one of the arrays a or b get exhausted, simply copy the rest of the other array.

```
void merge (int *a, int *b, int *c, int m, int n)
                                        // c is the destination array
{
     int i=0, j=0, k=0, p;

         // loop as long as neither array a nor array b is completed
     while ((i<m) && (j<n)) {
              if (a[i] < b[j])
              { c[k] = a[i]; i++; }
              else
              { c[k] = b[j]; j++; }
              k++;
     }

     if (i == m) {  // array a completed; copy rest of array b to array c
              for (p=j; p<n; p++)
              { c[k] = b[p]; k++; }
     } else {       // array b completed; copy rest of array a to array c
              for (p=i; p<m; p++)
              { c[k] = a[p]; k++; }
     }
}
```

Example: showing the merge phase only

Initial array A contains 16 elements:

- `66, 33, 40, 22, 55, 88, 60, 11, 80, 20, 50, 44, 77, 30, 47, 23`

Pass 1 :: Merge each pair of elements

- `(33, 66) (22, 40) (55, 88) (11, 60) (20, 80) (44, 50) (30, 70) (23, 47)`

Pass 2 :: Merge each pair of pairs

- `(22, 33, 40, 66)  (11, 55, 60, 88)  (20, 44, 50, 80)  (23, 30, 47, 77)`

Pass 3 :: Merge each pair of sorted quadruplets

- `(11, 22, 33, 40, 55, 60, 66, 88)  (20, 23, 30, 44, 47, 50, 77, 80)`

Pass 4 :: Merge the two sorted subarrays to get the final list

- `(11, 20, 22, 23, 30, 33, 40, 44, 47, 50, 55, 60, 66, 77, 80, 88)`

```c
void merge_sort (int *A, int n)
{
  int i, j, k, m;
  int *B, *C;
  if (n > 1)  {
    k = n/2;   m = n - k;
    B = (int *) malloc (k * sizeof(int));
    C = (int *) malloc (m * sizeof(int));
    for (i=0; i<k; i++)
        B[i] = A[i];
    for (j=k; j<n; j++)
        C[j-k] = A[j];
    // B contains first half of A
    // C contains second half of A
     merge_sort (B, k);
     merge_sort (C, m);
     merge (B, C, A, k, m); // dest A
     free(B); free(C);
   }
}
```

```c
void merge (int *a, int *b, int *c, int m, int n)
{
   int i=0, j=0, k=0, p;

    while ((i < m) && (j < n))  {
        if (a[i] < b[j])
            { c[k] = a[i]; i++; }
        else
            { c[k] = b[j]; j++; }
        k++;
    }

    if (i == m) {
        for (p=j; p<n; p++)
           { c[k] = b[p]; k++; }
    } else  {
        for (p=i; p<m; p++)
           { c[k] = a[p]; k++; }
     }
}
```

Time complexity of merge sort

If n denotes the number of elements to be sorted, then the number of comparisons required in merge sort is approximately proportional to `n log n`.

We need extra storage space as we have to temporarily create space for the arrays B and C.

Practically best sorting method: Quicksort

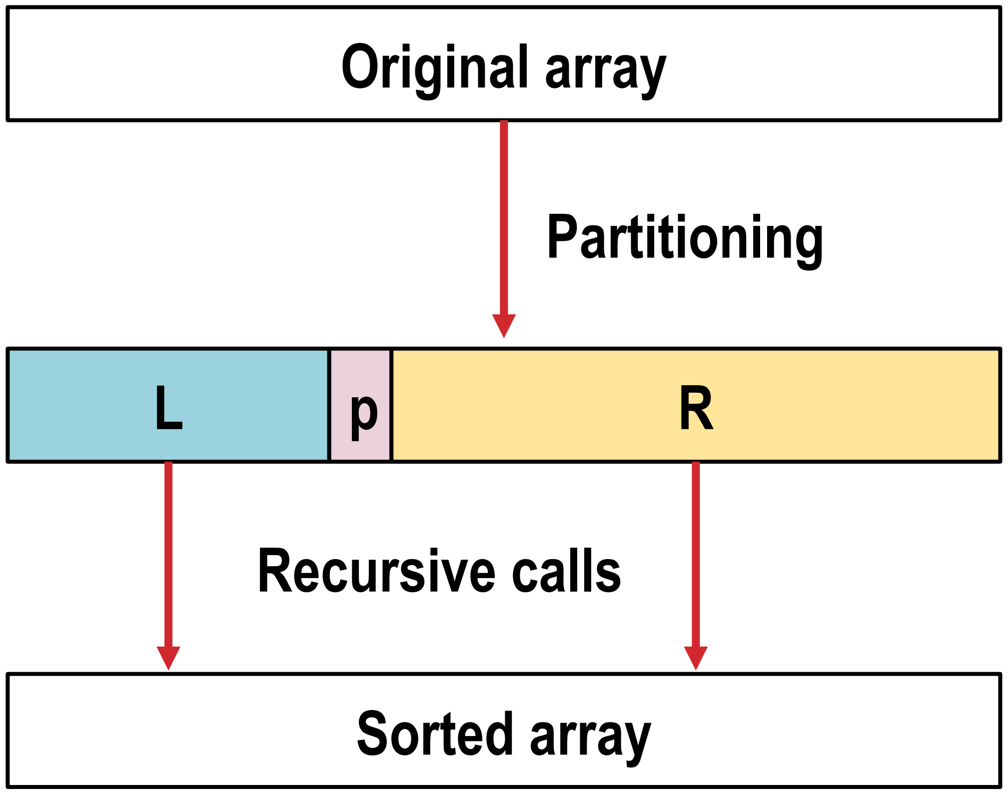
Introduction to Quick Sort

- Merge sort is a theoretically best (optimal) sorting algorithm.
- Quick sort is the practically best general-purpose sorting algorithm.
- Problems of merge sort:
    - Extra space requirement
    - Merging step is difficult to carry out without extra arrays.
- Quick sort is another recursive sorting algorithm.
- Quick sort takes a divide-and-conquer approach.
- In merge sort, the main work (merging) is done after the recursive calls return.
- In quick sort, the main work (partitioning) is done before the recursive calls are made.
- Basic idea of quick sort
    - Choose an element p of the array A as the pivot.
    - Decompose the array in three parts: L consisting the elements of A less than (or equal to) p, R consisting of the elements of A larger than p, and the single element p.
    - Recursively sort L and R.
    - Output sorted(L) followed by p followed by sorted(R).
    - If partitioning is done in A itself, then there is no task left after the recursive calls.

Quick sort: Skeleton of the algorithm

```
void quick_sort ( int A[], int n )
{
   int pivotidx;

   if (n <= 1) return;
   pivotidx = partition (A, n);
   quicksort (A, pivotidx);
   quicksort (A+pivotidx+1, n-pivotidx-1);
}
```



Partitioning using extra arrays

```
int partition ( int A[], int n )
{
   int *L, *R, p, i, j, l, r;

   if (n <= 1) return n-1;

   L = (int *)malloc(n * sizeof(int));
   R = (int *)malloc(n * sizeof(int));
   p = A[n-1];  // Choose the last element of A as pivot
   l = r = 0;   // Initialize the sizes of L and R
   for (i=0; i<=n-2; ++i)
      if (A[i] <= p) L[l++] = A[i]; else R[r++] = A[i];
   for (i=0; i<l; ++i) A[i] = L[i];     // Copy L to A
   A[i++] = p;                          // Append p to A
   for (j=0; j<r; ++j) A[i++] = R[j];   // Append R to A
   free(L); free(R);  // No further needs for L and R
   return l;
}
```

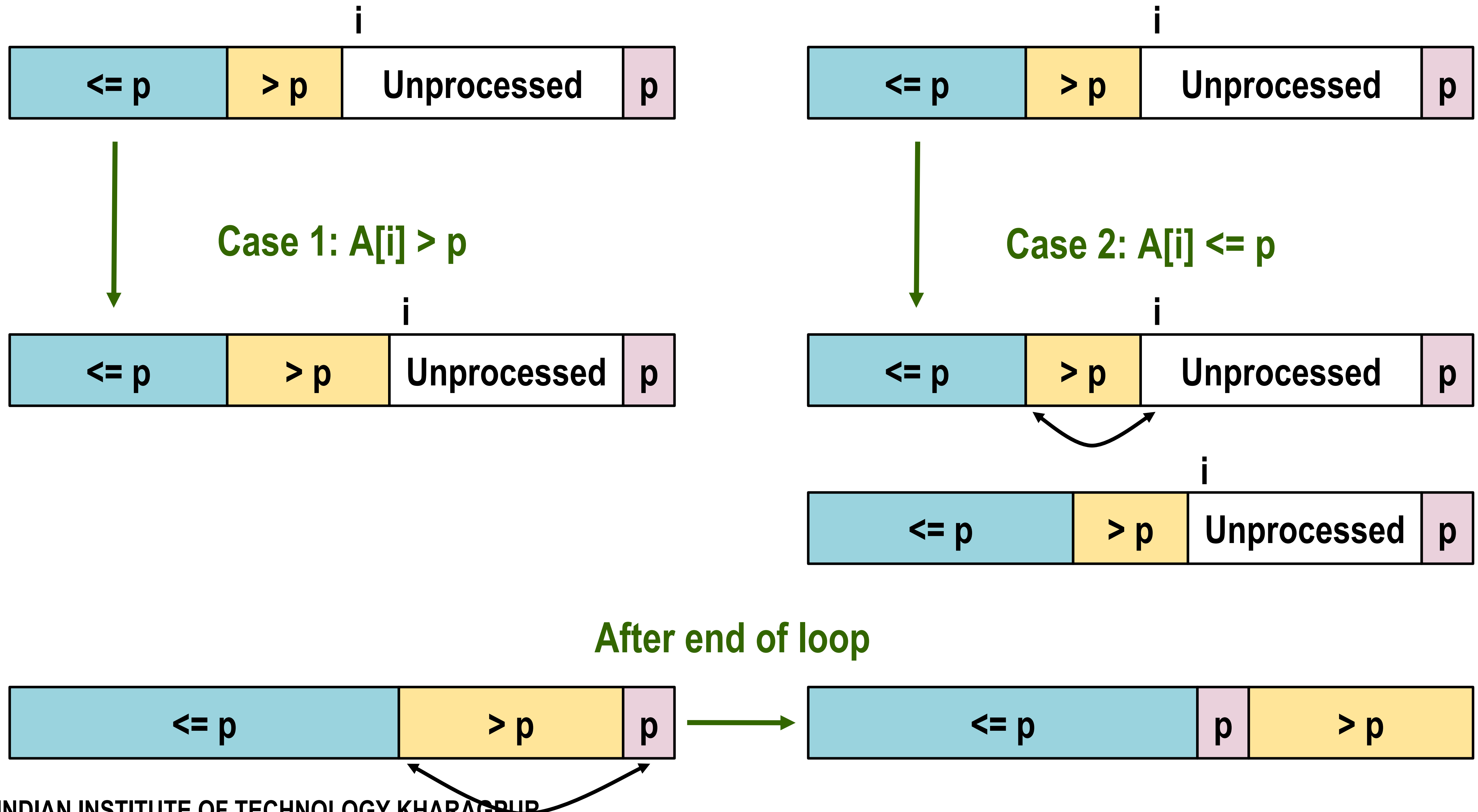
In-place partitioning

- Possibility of partitioning A without any extra arrays make quick sort attractive and efficient.
- There are many variants of the in-place partitioning algorithm.
- We follow the CLRS variant:

  Cormen , Leiserson, Rivest, and Stein, *Introduction to Algorithms*, 4th Edition, MIT Press

- We take p = A[n–1] as the pivot.
- The array A is always maintained as the concatenation LRUp, where
  - L consists of elements <= p
  - R consists of elements > p
  - U is the unprocessed part (elements in U are not yet classified to go to L or R)
- Each iteration processes one element from U, and sends that element to L or R as appropriate.
- After n – 1 iterations, there are no unprocessed elements, so the array is of the form LRp.
- It is then converted to the form LpR.
- Blocks (L and R) are never fully shifted. Only element swaps are used.
- This may destroy the order of the (equal) keys in the partitioned array.

In-place partitioning



In-place partitioning: The code

```
int partition ( int A[], int n )
{
   int lend = -1, i;
   int p, t;

   p = A[n-1];  // Last element of A is the pivot
   for (i=0; i<=n-2; ++i) {
      if (A[i] <= p) { // Region L grows
         ++lend;
         t = A[lend]; L[lend] = L[i]; L[i] = t;
      }
      // else Region R grows, ++i will do it
   }
   i = lend + 1;  // i is the first index of Region R
   t = A[i]; A[i] = A[n-1]; A[n-1] = t;
   return i;
}
```

In-place partitioning: An example

lend	*i*									
	10	21	8	3	15	27	9	16	5	15
−1	0	1	2	3	4	5	6	7	8	9

lend	*i*								
10	21	8	3	15	27	9	16	5	15
0	1	2	3	4	5	6	7	8	9

	lend	*i*							
10	8	21	3	15	27	9	16	5	15
0	1	2	3	4	5	6	7	8	9

		lend	*i*						
10	8	3	21	15	27	9	16	5	15
0	1	2	3	4	5	6	7	8	9

			lend	*i*					
10	8	3	15	21	27	9	16	5	15
0	1	2	3	4	5	6	7	8	9

			lend		*i*				
10	8	3	15	21	27	9	16	5	15
0	1	2	3	4	5	6	7	8	9

				lend		*i*			
10	8	3	15	9	27	21	16	5	15
0	1	2	3	4	5	6	7	8	9

				lend			*i*		
10	8	3	15	9	27	21	16	5	15
0	1	2	3	4	5	6	7	8	9

					lend			*i*	
10	8	3	15	9	5	21	16	27	15
0	1	2	3	4	5	6	7	8	9

					lend	*i*			
10	8	3	15	9	5	15	16	27	21
0	1	2	3	4	5	6	7	8	9

Performance of quick sort

- Running times are specified as "*roughly proportional to a function of the input size.*"
- No (comparison-based) sorting algorithm can run faster than n log n is the worst case.
- For merge sort:
    - All cases are the same. No specific best / worst / average case.
    - Each case has running time n log n for merge sort.
- For quick sort:
    - Best case: Partitioning divides the array roughly into two equal halves
    - Worst case: Partitioning always gives one subarray of size one less than the array.
    - Average case: The pivot is any one element (smallest to largest) with equal probability.
- Example of worst case: The array is already sorted in ascending or descending order.
- Running time of quick sort:
    - Best and average case: n log n
    - Worst case: n^2
- Quick sort is not theoretically optimal.
- In practice, quick sort is considered the fastest sorting algorithm for "general" arrays.