

Induction on Strings

7th Jan 2019

Instruction : Write the answers to the problems neatly in loose sheets with your name and roll number. Submit to the TA at the end of the class.

1. A palindrome can be defined as a string that reads the same forward and backward, or by the following definition.
 - (a) ϵ is a palindrome.
 - (b) If a is any symbol, then the string a is a palindrome.
 - (c) If a is any symbol and x is a palindrome, then axa is a palindrome.
 - (d) Nothing is a palindrome unless it follows from (a) through (c).

Prove by induction that the two definitions are equivalent.

2. The strings of balanced parenthesis can be defined in at least two ways.
 - (a) A string w over alphabet $\{(,)\}$ is balanced if and only if:
 - i. w has an equal number of ('s as)'s, and
 - ii. any prefix of w has at least as many ('s as)'s.
 - (b)
 - i. ϵ is balanced.
 - ii. If w is a balanced string, then (w) is balanced.
 - iii. If w and x are balanced strings, then so is wx .
 - iv. Nothing else is a balanced string.

Prove by induction on the length of a string that definitions (a) and (b) define the same class of strings.

3. Prove that any equivalence relation R on a set S partitions S into disjoint equivalence classes.
4. Show that the following are equivalence relations and give their equivalence classes.
 - (a) R_1 on integers $\rightarrow iR_1j$ iff $i = j$.
 - (b) R_2 on people $\rightarrow pR_2q$ iff p and q were born on the same hour of same day of some year.