CS21004 - Tutorial 8

Solution Sketch

1. Show that following language is not context-free using pumping lemma

   (a) $L_1 = \{a^{n!} : n \geq 0\}$
   Hints: Given the opponent's choice for m (Pumping lemma constant), we pick $a^{m!}(= uvxyz)$. Obviously, whatever the decomposition is, it must be of the form $v = a^k$, $y = a^l$. Then $w_0 = uxz$ (pump down) has length $m! - (k+l)$. This string is in L only if $m! - (k + l) = j!$ for some j. But this is impossible, since with $k + l \leq m$, $m! - (k + l) > (m - 1)!$. Therefore, the language is not context-free.

   (b) $L_2 = \{wtw^R | w, t \in \{0, 1\}^*\}$ and $|w| = |t|\}$
   Hints:
   Suppose on the contrary that A is context-free. Then, let p be the pumping length for A, such that any string in A of length at least p will satisfy the pumping lemma. Now, we select a string s in A with $s = 0^{2p}1^p0^p0^{2p}$.
   For s to satisfy the pumping lemma, there is a way that s can be written as $uvxyz$, with $|vxy| \leq p$ and $|vy| \geq 1$, and for any i, uv^ixy^iz is a string in A. There are only three cases to write s with the above conditions:

      i. **Case 1:** vy contains only 0s and these 0s are chosen from the last 0^{2p} of s. Let i be a number with $7p > |vy| \times (i + 1) \geq 6p$. Then, either the length of uv^ixy^iz is not a multiple of 3, or this string is of the form wtw' such that $|w| = |t| = |w'|$ with w' is all 0s and w is not all 0s (that is, $w' \neq w$).

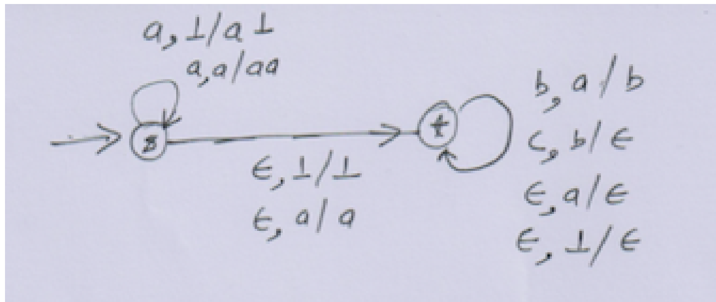
      ii. **Case 2:** vy does not contain any 0s in the last 0^{2p} of s. Then either the length of uv^2xy^2z is not a multiple of 3, or this string is of the form wtw' such that $|w| = |t| = |w'|$ with w' is all 0s and w is not all 0s (that is, $w' \neq w$).

      iii. **Case 3:** vy is not all 0s and some 0s are from the last 0^{2p} of s. As, $|vxy| \leq p$, vxy in this case must be a substring of 1^p0^p. Then either the length of uv^2xy^2z is not a multiple of 3, or this string is of the form wtw' such that $|w| = |t| = |w'|$ with w' is all 0s and w is not all 0s (that is, $w' \neq w$).

   In summary, we observe that there is no way s can satisfy the pumping lemma. Thus, a contradiction occurs and we conclude that A is not a context-free language.

2. Design NPDA for the following languages

   (a) $L_3 = \{a^i(bc)^j | i, j \geq 0, i \geq j\}$ – Give a PDA with 2 states (To Submit)

   (b) $L_4 = \{a^n b^m | n \neq m\}$
   Hints: $Q = \{q_0, q_1, q_2\}$, $\Sigma = \{a, b\}$, $\Gamma = \{a, z\}$, $F = \{q_2\}$ The transition function can be visualized as having several parts: a set to push a on the stack -
   $\delta(q_0, a, z) = \{(q_0, az)\}$, $\delta(q_0, a, a) = \{(q_0, aa)\}$
   a set to pop a on reading b, where the NPDA switches from state q_0 to q_1 -
   $\delta(q_0, b, a) = \{(q_1, \epsilon)\}$, $\delta(q_1, b, a) = \{(q_1, \epsilon)\}$
   a set to ensure $m \neq n$, where NPDA switches from state q_1 to q_2
   $\delta(q_1, b, z) = \{(q_2, z)\}$, $\delta(q_1, \epsilon, a) = \{(q_2, \epsilon)\}$
   and finally $\delta(q_2, \epsilon, z) = \{(q_2, \epsilon)\}$

3. Construct a NPDA that accepts the language generated by a grammar with productions: $S \rightarrow aSbb | a$

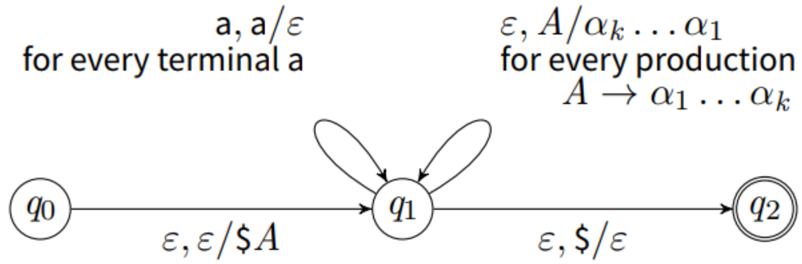
   Hints: From NPDA to CFG conversion rules, the final NPDA contructed contains 3 states $= \{q_0, q_1, q_2\}$ with final state being q_2.
   For every terminal, we have $\delta(q_1, a, a) = \{(q_1, \epsilon)\}$, $\delta(q_1, b, b) = \{(q_1, \epsilon)\}$

   For every production, $\delta(q_1, \epsilon, S) = \{(q_1, bbSa)\}$, $\delta(q_1, \epsilon, S) = \{(q_1, a)\}$

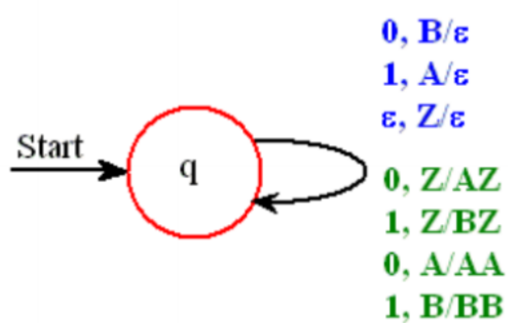
   Start Transition : $\delta(q_0, \epsilon, \epsilon) = \{(q_1, \$S)\}$,

   Accepting Transition : $\delta(q_1, \epsilon, \$) = \{(q_2, \epsilon)\}$,

   The general rule for CFG to PDA conversion is :



4. Consider the PDA $P = (\{q\}, \{0,1\}, \{Z, A, B\}, \delta, q, Z, \phi)$, where the transitions are shown in the following figure. Convert this PDA to CFG as per PDA-to-CFG conversion.



Hints:
$(q, q, Z) \to 0(q, q, A)(q, q, Z)$
$(q, q, Z) \to 1(q, q, B)(q, q, Z)$
$(q, q, A) \to 0(q, q, A)(q, q, A)$
$(q, q, B) \to 1(q, q, B)(q, q, B)$
$(q, q, B) \to 0$
$(q, q, A) \to 1$
$(q, q, Z) \to \epsilon$
with (q, q, Z) being the start variable.