CS21004 - Tutorial 6

Solution Sketch

Instructions: For the problems with (To submit), please write the answers neatly in loose sheets and submit to the TA before the end of the tutorial.

1. Provide Context Free Grammars (CFGs) for the following languages:

   a. $L_1 = \{ww^R | w \in \{0,1\}^*\}$

   $S \to A$

   $A \to 0A0 \mid 1A1 \mid \epsilon$

   b. $L_2 = \{a^i b^j c^k | i, j, k \geq 0$ and $i = j$ or $j = k\}$

   $S \to AX_1 \mid X_2B$

   $A \to aAb \mid \epsilon$

   $X_1 \to cX_1 \mid \epsilon$

   $B \to bBc \mid \epsilon$

   $X_2 \to aX_2 \mid \epsilon$

   c. $L_3 = \{a^{i_1} b^{i_1} a^{i_2} b^{i_2} \ldots a^{i_n} b^{i_n} | n, i_1, i_2, \ldots, i_n \geq 0\}$

   $S \to AS \mid \epsilon$

   $A \to aAb \mid \epsilon$

   d. $L_4 = \{0^i 1^j 2^k | k \leq i$ or $k \leq j\}$

$S \to S_1 \mid S_2$

$S_1 \to 0S_1 \mid 0S_12 \mid C$ $[k \le i]$

$C \to 1C \mid \epsilon$

$S_2 \to 0S_2 \mid S_3$

$S_3 \to 1S_3 \mid 1S_32 \mid \epsilon$ $[k <= j]$

2. Use Myhill-Nerode theorem to prove non-regularity for the following languages:

   a. L_5, where L_5 is the language of palindromes over $\{a, b\}$

   Solution: Assume that L_5 is regular, thus there is a Myhill Nerode relation $\equiv$. Let $k \neq m$ and $a^k \equiv a^m$. Now, by right congruence, $a^kba^k \equiv a^mba^k$. But this is not possible because $a^kba^k \in L_5$ but $a^mba^k \notin L_5$. Hence a contradiction. Thus, there are infinitely many equivalence classes for $\{a^k | k \geq 0\}$. Hence $\equiv$ is not Myhill-nerode and L_5 is non-regular.

   b. $L_6 = \{uu^Rv | u, v \in \Sigma^+\}$

   Solution: Assume that L_6 is regular, thus there is a Myhill Nerode relation $\equiv$. Let $k \neq m$ and $ab^{2k+1}a \equiv ab^{2m+1}a$. By right congruence $ab^{2k+1}aab^{2k+1}aa \equiv ab^{2m+1}aab^{2k+1}aa$. Note that the first string is in L_6 (take $u = ab^{2k+1}$ and $v = a$) but the second string cannot be because any prefix of the second string has to end with ba to qualify for uu^R. It happens for $ab^{2m+1}a$, which is not valid since it is not even, and $ab^{2m+1}aab^{2k+1}a$, which is not a palindrome. Hence, there are infinite equivalence classes for $\{ab^{2k+1}a | k \geq 0\}$