CS21004 - Tutorial 2

Solutions

Instructions: For the problems with (To submit), please write the answers neatly in loose sheets and submit to the TA before the end of the tutorial.

1. Consider the following two languages over the alphabet $\Sigma = \{a, b\}$

$$L_1 = \{a^n : n \geq 1\}$$
$$L_2 = \{b^n : n \geq 1\}$$

Describe the following languages as per the set notations (e.g., as above) as well as the precise definitions in English (e.g., L_1 can be defined as the set of all strings that have one or more a's but no b's).

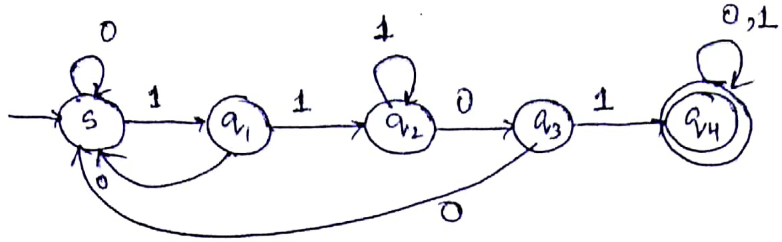
- $L_3 = \overline{L_1}$
  $\Rightarrow L_3 = \overline{L_1} = \{\epsilon\} \cup \{w : w \text{ includes at least one } b\}$
- $L_4 = (L_1 L_2)^+$ (To submit)
  $\Rightarrow L_4 = (L_1 L_2)^+ = \{awb : w \text{ is any string on } \Sigma\}$

2. Let $\Sigma = \{0, 1\}$. Give DFA's accepting the following strings

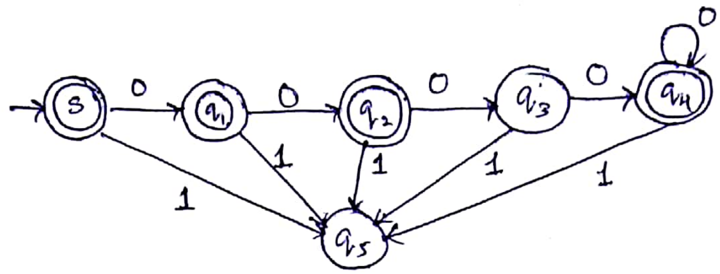
   (a) The set of all strings containing 1101 as substring
   (b) $\{0^n | n \geq 0, n \neq 3\}$
   (c) The set of all strings beginning with 101 (To submit)
   (d) The set of all strings, which are divisible by 5. (To submit)
   (e) $\{01^4 x 1^3 | x \in \{0, 1\}^*\}$ (To submit)

2.

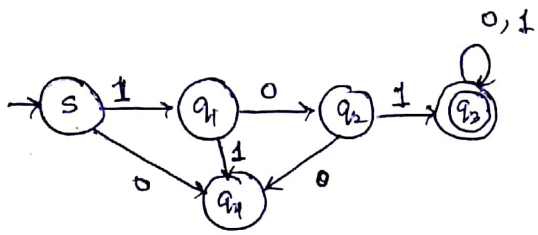
(a)


s
q1
q2
q3
q4
0
1
0,1

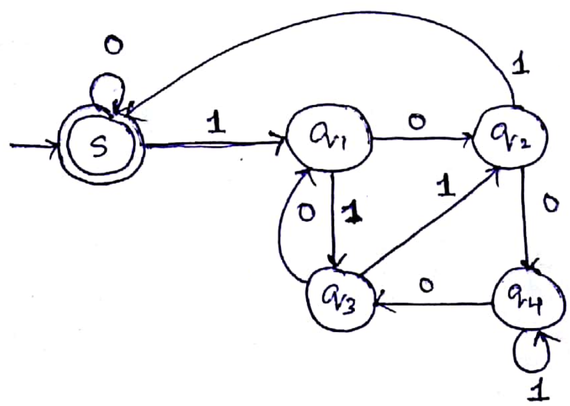
(b)


s
q1
q2
q3
q4
q5
0
1

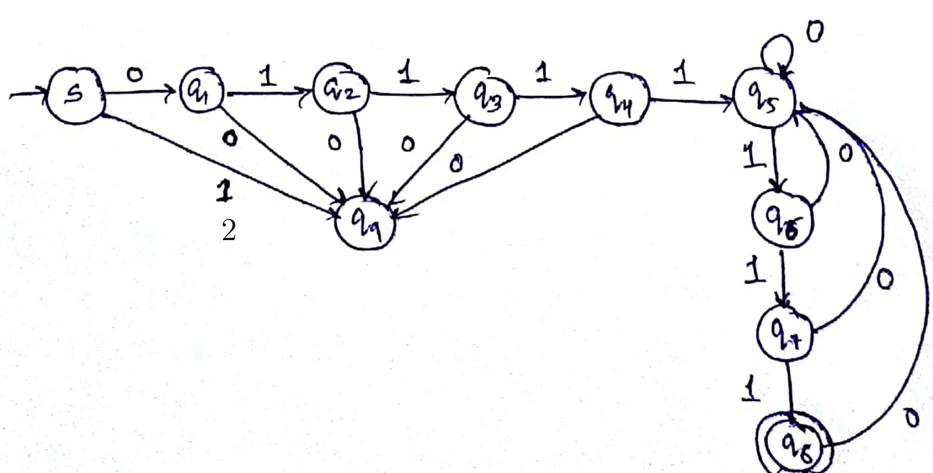
(c)


s
q1
q2
q3
q4
0
1
0, 1

(d)


s
q1
q2
q3
q4
0
1

(e)


s
q1
q2
q3
q4
q5
q6
q7
q8
q9
0
1

3. For any language L over Σ, the *prefix closure* of L is defined as

$$Pre(L) = \{x \in \Sigma^* \mid \exists y \in \Sigma^* \text{ such that } xy \in L\}$$

Prove that if L is regular then so is $Pre(L)$. (To submit)

$\Rightarrow$ Suppose L is recognized by a DFA D. We need to build a new DFA D' from D. D' has exact same states as D, but for each state s of D, if there's an accepting state reachable from s, then the corresponding state in D' will be an accepting state. And D' will recognize $Pre(L)$.

4. Prove that $\forall L_1, L_2$, $(L_1L_2)^R = L_2^R L_1^R$. (Home)

$\Rightarrow$ Let $\sigma \in (L_1L_2)^R$. Hence $\sigma^R \in L_1L_2$. Hence $\sigma^R = xy$ such that $x \in L_1$ and $y \in L_2$. Now $\sigma = (xy)^R = y^R x^R \in L_2^R L_1^R$.

Let $\sigma \in L_2^R L_1^R$. Hence $\sigma = xy$ such that $x \in L_2^R$ and $y \in L_1^R$. Then $x^R \in L_2$ and $y^R \in L_1$. Hence $\sigma^R = y^R x^R \in L_1L_2$. Hence $\sigma \in (L_1L_2)^R$. Thus proved.

5. Construct a DFA for the set of all strings over the alphabet $\{0, 1\}$ that, when interpreted in reverse as a binary integer are divisible by 5. Example of strings in this language are 0, 10011, 1001100, 101, while strings like 111 are not in the language. Note, 10011 = 25, when interpreted in reverse as a binary integer. (Home)

$\Rightarrow$ In this specific case, you can just reverse all the transitions in 2(d) and the resulting DFA would work.