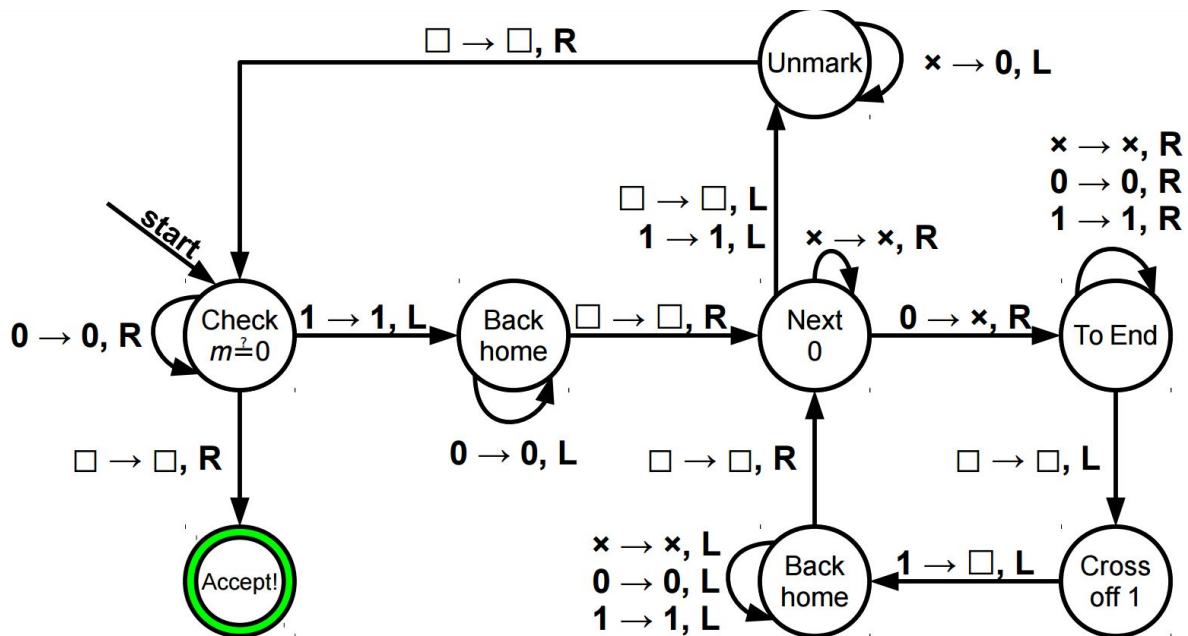


Solution:

1) M that starts with $0^m 1 0^n$ and halts with $0^{\max\{m-n, 0\}}$ on tape

- M repeatedly replaces leading 0 by blank, then searches right for a 1 followed by 0 and changes 0 to 1.
- Then, M moves left until it encounters a blank and repeats this cycle.
- The repetition ends if
 - Searching right for 0, M encounters a blank. Then all n 0's in $0^m 1 0^n$ have been changed to 1 and $n + 1$ of m 0's changed to blank. M replaces $n + 1$ 1's by a 0 and n blanks leaving $m - n$ 0's on the tape.
 - Beginning the cycle, M cannot find a 0 to change to a blank, because first m 0's have already been changed. Then $n \geq m$, so $\max\{m - n, 0\} = 0$. M replaces remaining 1's, 0's by blanks

2) $L = \{0^n 1^m \mid n, m \in \mathbb{N} \text{ and } m \text{ is a multiple of } n\}$



3)

- $L = \{a^m b^n c^m d^n \mid m, n \geq 1\}$
 $S \rightarrow AB$
 $A \rightarrow aAX \mid aX$
 $B \rightarrow bBd \mid bYd$
 $Xb \rightarrow bX$
 $XY \rightarrow Yc$

$Y \rightarrow \epsilon$

- b) $L = \{a^i b^j c^k \mid 1 \leq i \leq j \leq k\}$
 $S \rightarrow aS'bX \mid abX$
 $S' \rightarrow aS'bC \mid S'bC \mid S'C \mid bC \mid C$
 $Cb \rightarrow bC$
 $CX \rightarrow Xc$
 $X \rightarrow c$

4) If L is recursively enumerable and L' is recursively enumerable, then L is recursive.

- Let M and M' be TMs that accept L and L', respectively.
- Run M and M' simultaneously.
- For any word x, it must be accepted by one of M or M'
- So, either M or M' will halt and accept
- If M halts and accepts then halt and accept
- If M' halts and accepts then halt and reject

The TM that runs M and M' simultaneously always halts and accepts or rejects so it decides L and L is recursive.

5) Show that a Multi-head Turing Machine M can be simulated by a Single-head Turing Machine M'

- Let M have k heads. Then M' will have $(k+1)$ tracks on a single tape.
- One track will contain the contents of the tape of M and the other tracks are used to mark the head positions.
- One move of M is simulated by M' by making a left to right sweep followed by a right to left sweep.
- One fact about which one has to be careful here is the time when 2 heads scan the same symbol and try to change it differently. In this case, some priority among heads has to be used.