Q1 (a) $L = L_1 \cup L_2$

L_1: w has odd length

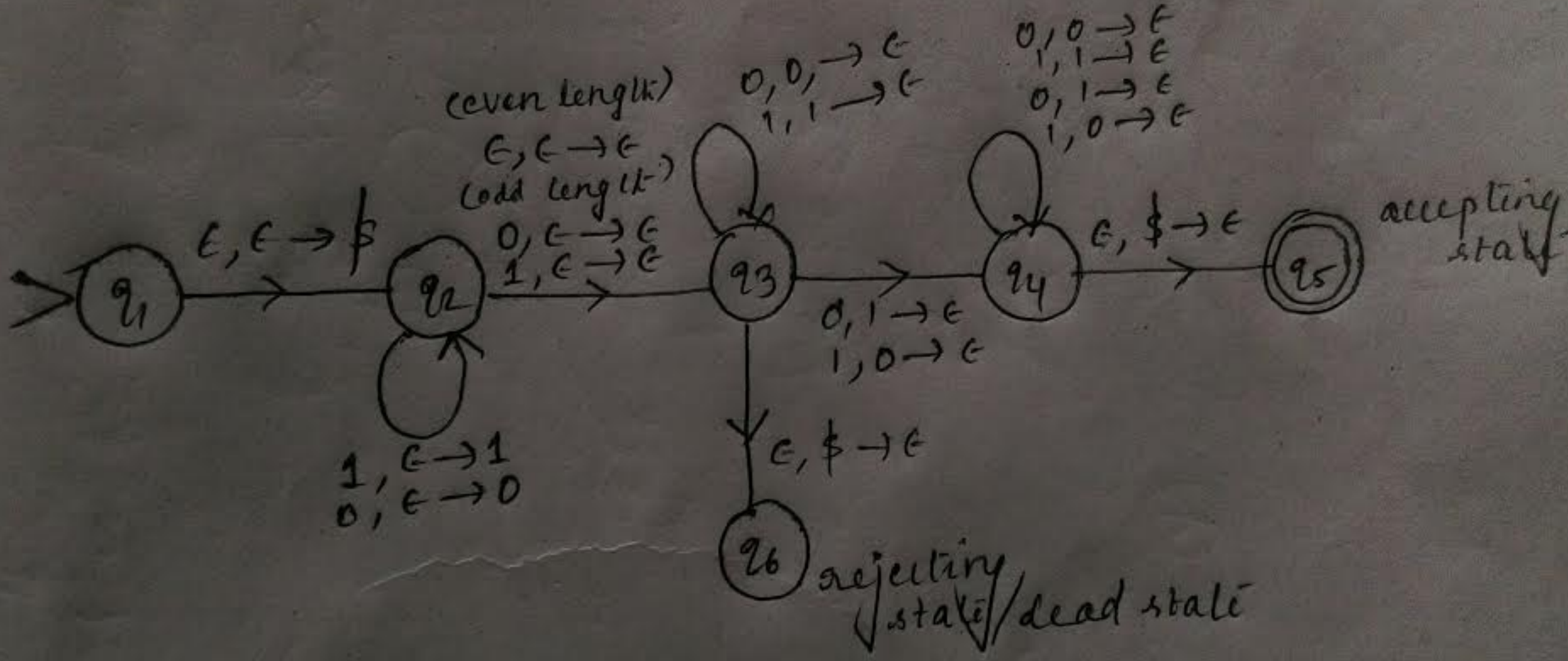
L_2: First half of w doesn't equal second half

$\Rightarrow L_2 = \{x_1 \sigma_1 y_1 x_2 \sigma_2 y_2 \mid x_1, y_1, x_2, y_2 \in \{a,b\}^*, |y_1 x_2| = |x_1| + |y$

(b) $L = L_1 \cup L_2$

L_1: $m < 2n$

L_2: $2n < 3m$


ϵ, ϵ → $
(even length)
ϵ, ϵ → ϵ
(odd length)
0, ϵ → ϵ
1, ϵ → ϵ
0, 0 → ϵ
1, 1 → ϵ
0, 0 → ϵ
1, 1 → ϵ
0, 1 → ϵ
1, 0 → ϵ
0, 1 → ϵ
1, 0 → ϵ
ϵ, $ → ϵ
1, ϵ → 1
0, ϵ → 0
ϵ, $ → ϵ
q1
q2
q3
q4
q5
q6
accepting state
rejecting state/dead state

Q3 (a) $\hat{L} = L \cap \{\Sigma^2\}^*$

Since $\{\Sigma^2\}^*$ is a Regular Language

CFG → closed under intersection with Reg. Language

$\Rightarrow \hat{L} \rightarrow$ CFG.

(b) Say M: Σ^* - {all strings over Σ with length at most 10}

⇓

Regular Language

$\Rightarrow$ M is also regular [closed under complementation]

Again, $\hat{L} = L \cap M$

$\Rightarrow \hat{L} \rightarrow$ CFG.

We have seen that every CFG can be converted to an equivalent PDA M_1 that accepts by empty stack. M_1 has only 1 state.

We've also seen that every PDA M_1 that accepts by empty stack is equivalent to another PDA M_2 that accepts by final state.

However, M_2 should have two loose states than M_1. $\Rightarrow$ 3 states.

$M_1 = (\{q\}, \Sigma, \Gamma, \delta, q, \perp, \phi)$

To achieve an M_2 with 2 states, we use some new stack symbols

$$M_2 = (\{q, q_f\}, \Sigma, \Gamma \cup \Gamma', \delta', q, \perp', \{q_f\})$$

where $\Gamma' = \{Z' \mid Z \in \Gamma\}$

i.e. for all pushdown symbols, we use a primed version as well.

δ': We retain the original transitions except that we add another transition corresponding to the primed version

If $\delta(q, a, A) = (q, B_1 \ldots B_k)$

then i) $\delta'(q, a, A) = (q, B_1 \ldots B_k)$ <- [If A is not the last symbol on the stack, this transition is used]

ii) $\delta'(q, a, A') = (t, B_1 \ldots B_k')$

↖ when A' is at bottom

If $\delta(q, a, A) = (q, \epsilon)$

then i) $\delta'(q, a, A) = (q, \epsilon)$

ii) $\delta'(q, a, A') = (q_f, \epsilon)$

For all stack symbols not on the bottom, it behaves as M_1, but a different behavior when a stack symbol is at the bottom.