CS21004 - Tutorial 1

January 8th, 2016

1. Consider the following two languages over the alphabet $\Sigma = \{a, b\}$

$$L_1 = \{a^n : n \geq 1\}$$
$$L_2 = \{b^n : n \geq 1\}$$

Describe the following languages as per the set notations (e.g., as above) as well as the precise definitions in English (e.g., L_1 can be defined as the set of all strings that have one or more a's but no b's).

- $L_3 = {L_1}^*$
- $L_4 = \overline{L_1}$
- $L_5 = L_1 \cup L_2$
- $L_6 = L_1 L_2$
- $L_7 = ({L_1}^2)({L_2}^2)({L_1}^2)$
- $L_8 = (L_1 \cup L_2)^*$
- $L_9 = (L_1 L_2)^+$

2. Consider the alphabet $\Sigma = \{a, b\}$. Is there any language L over this alphabet for which $(\overline{L})^* = \overline{L^*}$? If yes, give an example of such a language; if no, explain why.

3. Let $\Sigma = \{a, b\}$, $L = w \in \Sigma^+$. Design an automaton that accepts the language L.

4. A language L is called regular if you can construct a DFA that accepts the language. Consider language L_1 over $\Sigma = \{a, b\}$, defined as
$L_1 = \{w \in \Sigma^* | w = (ab)^n, n \geq 1\}$
Is the language L_1 regular?

5. Let $\Sigma = \{0, 1\}$. Give DFA's accepting the following strings

- The set of all strings containing 1101 as substring
- The set of all strings beginning with 101
- The set of all strings that begin with 01 and end with 11
- The set of all strings, when interpreted *in reverse* as a binary integer, is divisible by 3.