

This exam contains 1 page and 3 problems.

You should *not* use your books, notes, or any calculator. Be *precise* in your answers. Intuitive justifications may not carry any marks, when you are asked to prove. All the *sub-parts* of a problem should be answered at *one place* only. On multiple attempts, *cross* any attempt that you do not want to be graded for.

There are no clarifications. In case of doubt, you can take a valid assumption, state that properly and continue.

1. (6 points) Let L denote the context free language $\{ww^R \mid w \in \{a, b\}^*\}$, where w^R denotes the reverse of w . Prove or disprove: \bar{L} is context free, where \bar{L} denotes the complement of L .
2. (7 points) Consider the following language. Prove or disprove that this is a CFL.

$$L_1 = \{xyx \mid x, y \in \{a, b\}^* \text{ and } |x| \geq 1\}$$

3. (7 points) Construct an unrestricted grammar for the following language:

$$L_2 = \{wc^m d^n \mid w \in \{a, b\}^* \text{ and } m = \text{number of } a\text{'s in } w \text{ and } n \text{ equals the number of } b\text{'s in } w\}$$