Formal Languages and Automata Theory **CS21004**
Spring Semester, 2016 **Class Test 1**
08/02/16 **Time Limit: 1 Hour**
Maximum Marks: 20

This exam contains 1 page and 4 problems.

You should *not* use your books, notes, or any calculator. Be *precise* in your answers. Intuitive justifications may not carry any marks, when you are asked to prove. All the *sub-parts* of a problem should be answered at *one place* only. On multiple attempts, *cross* any attempt that you do not want to be graded for.

There are no clarifications. In case of doubt, you can take a valid assumption, state that properly and continue.

1. (4 points) Let L_1 and L_2 be two infinite languages, defined over the alphabet $\{a, b\}$, satisfying $L_1 \cap L_2 = \phi$ and $L_1 L_2 = L_2 L_1$. If such a language pair exists, give an example. If not, you must prove it.

2. (5 points) Construct an NFA to accept the regular expression $b(((ba)^* + bbb)^* + a)^* b$, such that the number of states are as minimum as possible. You should not use ϵ-transitions. [Hint: the required states are ≤ 5. You start loosing marks as the number of states in your NFA increase beyond the required number.]

3. (5 points) Show that regular languages are closed under doubling. If language L is regular, then so also is the language L_2 defined as

$$L_2 = \{\text{two } x | x \in L\}$$

where string doubling (two) is defined inductively as

$$\text{two } \epsilon = \epsilon$$

$$\text{two } ax = aa \cdot (\text{two } x)$$

4. (6 points) Use the pumping lemma to prove that the following language is non-regular.

$$L = \{b^p a b^q | p, q \geq 0, |p - 2q| = r^2, r > 0\}$$