

Solution 1: It is not a CFL.

Assume that L is context-free. Then it satisfies the pumping lemma. Let n be the constant of that lemma. Now consider $u = xyx$ with $x = a^n b^n$ and $y = \Lambda$. Thus $u = a^n b^n a^n b^n$. There must exist words p, q, r, s, t such that $u = pqrst$ such that $|qs| > 0$, $|qrs| \leq n$, and $pq^i r s^i t \in L$ for every $i \geq 0$. We distinguish two cases:

1. qs consists only of a 's from the first group of a 's in u or it consists only of b 's from the second group of b 's. Then $pq^2 r s^2 t$ is either $a^{n+j} b^n a^n b^n$ or $a^n b^{n+j} a^n b^n$ for some $j \geq 1$, which are both not in L . A contradiction.
 2. qs contains a b from the first group of b 's in u or it contains an a from the second group of a 's in u . Then $pq^0 r s^0 t$ is either $a^k b^l a^m b^n$ or $a^n b^k a^l b^m$ with $k, m \geq 1$ and $l < n$. Neither of these words is in L , again a contradiction.
- Consequently, we always end up with a contradiction and so L is not a CFL.

Solutions 2:

L contains the strings of the following types

- i) All strings of odd length
- ii) All strings of even length such that for some i , i^{th} symbol and the i^{th} last symbol are different.

Strings corresponding to i) can be generated as

$$S_1 \rightarrow aaS_1 | bbS_1 | abS_1 | baS_1 | a | b$$

Strings corresponding to ii) can be generated as

$$S_2 \rightarrow aS_2 a | bS_2 b | T_2$$

$$T_2 \rightarrow aU_2 b | bU_2 a$$

$$U_2 \rightarrow aU_2 a | aU_2 b | bU_2 a | bU_2 b | \epsilon$$

Note that L contains ϵ , so L won't

$$S \rightarrow S_1 | S_2 \text{ will generate } L$$

T_2 above introduces at least one asymmetry.

Solution 3:

The idea here is to generate a c every time we generate an a and to generate a d every time we generate a b. We'll do this by generating the nonterminals C and D, which we will use to generate c's and d's once everything is in the right place. Once we've finished generating all the a's and b's we want, the next thing we need to do is to get

all the D's to the far right of the string, all the C's next, and then have the a's and b's left alone at the left. We guarantee that everything must line up that way by making sure that C can't become c and D can't become d unless things are right. To do this, we require that D can only become d if it's all the way to the right (i.e., it's followed by #) or it's got a d to its right. Similarly with C. We can do this with the following rules;

$S \rightarrow S_1\#$

$S_1 \rightarrow aS_1C$

$S_1 \rightarrow bS_1D$

$S_1 \rightarrow \epsilon$

$DC \rightarrow CD$

$D\# \rightarrow d$

$Dd \rightarrow dd$

$C\# \rightarrow c$

$Cd \rightarrow cd$

$Cc \rightarrow cc$

$\# \rightarrow \epsilon$

So with R as given above, the grammar $G = (\{S, S_1, C, D, \#, a, b, c, d\}, \{a, b, c, d\}, R, S)$