Class-Test -1 Solutions

Solution 1 : Yes, such a pair exists (many such pairs)
Example:
L1 = a*, L2 = a*ba*

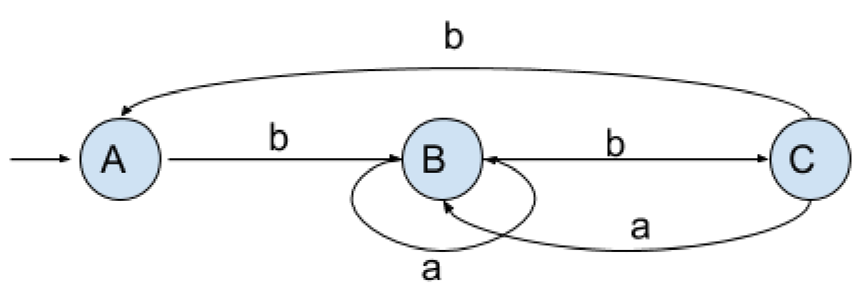
Solution 2: [5 marks for proving regularity under doubling, partial marks may be given]

Let L be regular. Then there is an NFA $N = (Q_N, \Sigma, \Delta_N, S_N, F_N)$ such that $L(N) = L$. Define another NFA, N_2, as follows: start with N and replace every edge $q \xrightarrow{a} q'$ with two edges $q \xrightarrow{a} q''$ and $q'' \xrightarrow{a} q'$ by inserting (for each edge!) a new state q''. We can formalize this idea by taking as states of N_2 the states of N plus the edges of N, the latter represented for example as the set of triples $\{(q_N, a, q'_N) \mid q_N \in Q_N,\ q'_N \in \Delta_N(q_N, a)\}$. So, we can define N_2 as follows:

- $Q_{N_2} \triangleq Q_N \cup \{(q_N, a, q'_N) \mid q_N \in Q_N,\ q'_N \in \Delta_N(q_N, a)\}$
- Δ_{N_2} is given by the two defining equations:
  $\Delta_{N_2}(q_N, a) \triangleq Q_N \cup \{(q_N, a, q'_N) \mid q'_N \in \Delta_N(q_N, a)\}$ and
  $\Delta_{N_2}((q_N, b, q'_N), a) \triangleq$ if $a = b$ then $\{q'_N\}$ else $\emptyset$
- $S_{N_2} \triangleq S_N$
- $F_{N_2} \triangleq F_N$

It is straightforward to show that, for the so constructed NFA, $L(N_2) = L_2$, thus implying that L_2 is regular. Hence, regular languages are closed under doubling.

Solution 3: Final state: C
[5 marks for this solution with 3 states, 4 marks for 4 state (correct) solution, 3 marks for 5 state (correct) solution, 0 marks otherwise]



Solution 4: Game with the Demon

Solution with a and b interchanged,
m and n, replaced with p and q

4. Game with the Demon

[2 marks for identifying steps
4 marks for correctness]

Step 1: The demon picks k.

Step 2: You pick

$$w = a^{2k+1} b a^k \in L$$

$$|w| > k$$

Let $x = a^{2k+1} b$, $y = a^k$, $z = \epsilon$

Step 3. Demon picks u, v, w

$$y = uvw$$

$$a^r a^s a^t$$

Step 4. You pick i

Now, for a general v & i,

$$w = a^{2k+1} b a^{k+(i-1)s}$$

$$|n - 2m| = |2k+1 - 2k - 2(i-1)s|$$

$$= |2(i-1)s - 1| \qquad |s| > 0$$

Let us take $i = 2s+1$

$$\Rightarrow |n - 2m| = 4s^2 - 1$$

Since $4s^2$ is a perfect square ≥ 4

$4s^2 - 1$ will not be a perfect square as the difference between 2 perfect squares ≥ 3