CS21201/CS21001 Discrete Structures, Autumn 2025–2026

Class Test 1

3rd–September–2025 | 06:30pm–07:30pm | Maximum marks: 30

Roll no: ____________ **Name:** ______________________________

[*Be brief and precise. Answer all questions.*]

1. How many binary strings of length n have exactly k transitions (indices i for string s with $s_i \neq s_{i+1}$)? Assume $0 \leq k \leq n-1$. **(4)**

Solution $2\binom{n-1}{k}$. Pick the k transition indices among the $n-1$ adjacent pairs. Once the indices are fixed, just choose the first bit - 0 or 1.

2. A bakery sells 6 types of pastries. A customer wants to buy 12 pastries in total. How many ways are there if the customer must buy at least one of each of the first two types and no more than 9 of any one type? **(6)**

Solution Number of solutions for $x_1 + x_2 \ldots + x_6 = 10$ is $\binom{15}{5}$ [Actual count for the first two is $x_1 + 1$ and $x_2 + 1$]. Out of these, there are **4 ways**, where one of 3rd - 6th pastries can be 10. For first pastry to be 10: either second is 2 (**1 way**) or 1 (**4 ways**, one with each of 3rd-6th being 1). Same with the second pastry (**5 ways**). Finally, there are **2 ways** where one of the first two pastries can be 11, and another 1. Final answer = $\binom{15}{5} - 16 = 2987$.

3. Formalise these statements and determine (with truth tables or otherwise) whether they are consistent (i.e. if there are some assumptions on the atomic propositions that make it true).

The system is in a multiuser state iff it is operating normally. If the system is operating normally, the kernel is functioning. Either the kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode.

Use these propositions: M, O, K, I. **(5)**

The system is in a multiuser state iff it is operating normally. If the system is operating normally, the kernel is functioning. Either the kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode.

Solution

Propositions:

m: system is in multiuser state

o: operating normally

k: kernel is functioning properly

i: system is in interrupt mode

Premises:

P1: $(m \to o) \land (o \to m)$

P2: $o \to k$

P3: $\neg k \lor i$

P4: $\neg m \to i$

P5: $\neg i$

The statements are not consistent

Contradiction using Rules of Inference

1. $\neg i$	(From P5)
2. $\neg m \to i$	(From P4)
3. $\neg i \to m$	(Contrapositive of 2)
4. m	(Modus Ponens of 1 and 3)
5. $m \to o$	(Conjunctive Simplification of P1)
6. o	(Modus Ponens of 4 and 5)
7. $o \to k$	(From P2)
8. k	(Modus Ponens of 7 and 8)
9. $\neg k \lor i$	(From P3)
10. $\neg i$	(Disjunctive Syllogism of 8 and 9)
11. F	(Conjunction of 1 and 10)

4. Using the principle of mathematical induction prove that $\forall n \in \mathbb{N}$, $f(n) = g(n)$, where

$$f(n) = 1 - \frac{1}{2} + \frac{1}{3} - \cdots + \frac{1}{2n+1} - \frac{1}{2n},\ g(n) = \frac{1}{n+1} + \cdots + \frac{1}{2n}.$$

(8)

Solution [Hint:] Show that $f(n+1) - f(n) = g(n+1) - g(n) \Rightarrow f(n+1) - g(n+1) = f(n) - g(n)$, then use PMI to prove the result.

5. Prove or disprove: There exist $a, b \in \mathbb{N}$ such that $a(a+1)(a+2) = b^2$. **(7)**

Solution [Hint:] Disprove: Consider the statement to be true. Observe that $GCD(a, a+1) = GCD(a+1, a+2) = 1$. Thus for all prime p, $p|(a+1) \to p \nmid a$ and $p \nmid (a+2)$. Now, for all such p, $a(a+1)(a+2) = b^2$ and $p|(a+1)$ implies $p|b^2$. Use this fact, to prove $(a+1)$ is a perfect square. Then argue that $a(a+2)$ must be a perfect square. But $a^2 < a(a+2) < a^2 + 2a + 1 = (a+1)^2$, thus $a(a+2)$ cannot be a perfect square (Contradiction).