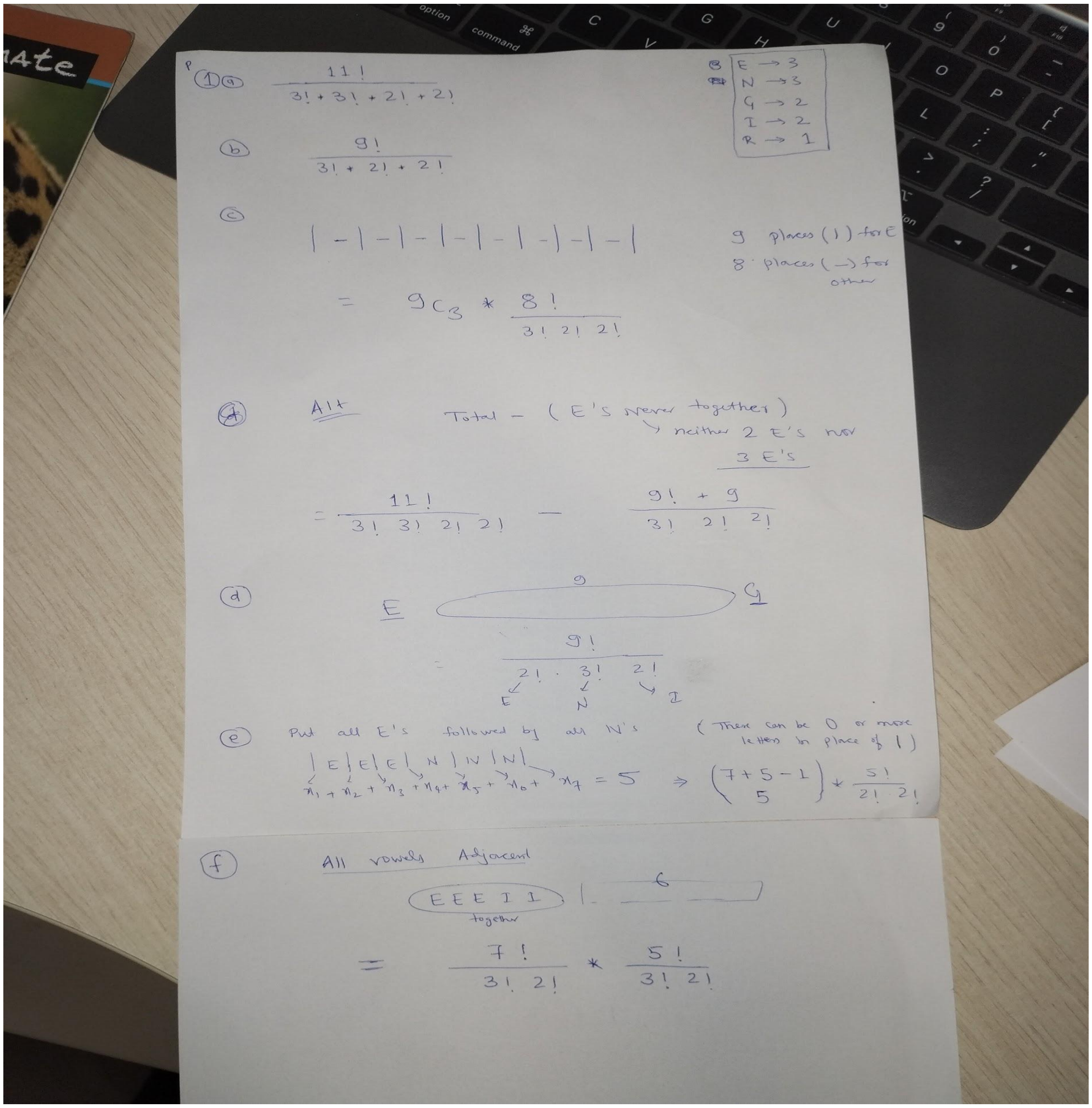
# CS21201 Discrete Structures

## Tutorial & Practice Problems Solution

## Elementary Counting Techniques

**Problem 1**


1 (a) $\frac{11!}{3! * 3! * 2! * 2!}$

E → 3
N → 3
G → 2
I → 2
R → 1

(b) $\frac{9!}{3! * 2! * 2!}$

(c) | − | − | − | − | − | − | − | − |

9 places ( | ) for E
8 places ( — ) for other

$= {}^9C_3 * \frac{8!}{3!\ 2!\ 2!}$

Alt: Total − ( E's never together )
neither 2 E's nor 3 E's

$= \frac{11!}{3!\ 3!\ 2!\ 2!} - \frac{9! + 9}{3!\ 2!\ 2!}$

(d) E ___ 9 ___ G

$= \frac{9!}{2! \cdot 3! \cdot 2!}$ (E, N, I)

(e) Put all E's followed by all N's ( There can be 0 or more letters in place of | )

| E | E | E | N | N | N |

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 5 \Rightarrow \binom{7+5-1}{5} * \frac{5!}{2! \cdot 2!}$

(f) All vowels Adjacent

EEEII ___ 6 ___
together

$= \frac{7!}{3!\ 2!} * \frac{5!}{3!\ 2!}$

**Problem 2**

Proceed as in the derivation of Catalan numbers. The answer is $\binom{m+n}{n} - \binom{m+n}{n-1} = \frac{m-n+1}{m+1}\binom{m+n}{n}$.

**Problem 4**

P 4 (a) 50 objects (people)

take 5 ( first five runner position)

$$= P(50,5) = \frac{50!}{45!}$$

(b) For top 3 runners

Krishna & Shyam can finish in $3! = 6$ ways.

For each of these 6 ways, there are $P(48,3)$ ways for other 3 finishers. [For top 5]

Hence, by product rule

Ways to award trophies with Shyam & Krishna among top 3 $= 6 * P(48,3)$

**Problem 5**

P5

□ □ □ □ □ □ □

3 choices (6, 7, 9)

If first digit is "6"

Remaining digit can be placed in any order, but 5 is repeated twice

$\Rightarrow \dfrac{6!}{2!} = 360$

If first digit is "7 or 9"

In this case 5 as well as 6 is repeated twice

$\Rightarrow 2 * \dfrac{6!}{2! \cdot 2!} = 360$

Total $= 360 + 360 = 720$

**Problem 6**

P6

Committee $\boxed{18}$

Men: 15
Women: 20

(a) $\binom{15+20}{18} = \binom{35}{18}$

(b) $\binom{15}{8}\cdot\binom{20}{6}\cdot\binom{21}{4}$

$15-8=7$
$20-6=14$
$8+6=14$

(c) No of Men

0, 2, 4, 6, 8, 10, 12, 14

$$\binom{15}{0}\cdot\binom{20}{18}+\binom{15}{2}\cdot\binom{20}{16}+\binom{15}{4}\cdot\binom{20}{14}$$

$$+\cdots+\binom{15}{14}\binom{20}{4}$$

$$=\sum_{i=0}^{7}\binom{15}{2i}\cdot\binom{20}{18-2i}$$

(d) More men than women

No of Men: 10, 11, 12, 13, 14, 15.

$$\binom{15}{10}\cdot\binom{20}{8}+\binom{15}{11}\cdot\binom{20}{7}+\cdots+\binom{15}{15}\cdot\binom{20}{3}$$

$$=\sum_{i=10}^{15}\binom{15}{i}\cdot\binom{20}{18-i}$$

(e) at least 8 men $\sum_{i=8}^{15}\binom{15}{i}\cdot\binom{20}{18-i}$

Total

**Problem 7**

P ⑦ (a) Treat all b balls as single object.

$$= \frac{(r+g+1)!}{r! \cdot g!}$$

(b) Green & red balls can be arranged in $\frac{(g+r)!}{g! \cdot r!}$ ways

Now, Insert b blue balls in bet^n or at the left-right ends without repetition.

$$= \frac{(g+r)!}{g! \cdot r!} \cdot \binom{g+r+1}{b}$$

(c) No blue after green

Put all blue balls followed by green balls → (1) way

Now inserting red balls at in bet^n or at the both ends with repetition allowed.

$$= \binom{b+g+1 + r - 1}{r} = \binom{r+g+b}{r}$$

(d) Start with Blue end with Non-Blue

Put a Blue ball at begining & a red/green at end.

$$\text{count} = \frac{(b-1 + g + r-1)!}{(b-1)!\ g!\ (r-1)!} + \frac{(b-1 + r + g-1)}{(b-1)! \cdot r! + (g-1)!}$$

$$= \frac{(r+g+b-2)!}{(r-1)!\,(g-1)!\,(b-1)!} \cdot \left( \frac{1}{g} + \frac{1}{r} \right)$$

**Problem 8**

P8

For '19'

There must be an odd no of 1's (betn 1 & 19 inclusive)

For $(2K+1)$ 1's $0 \le K \le 9$, there are $(2K+2)$ locations to select with repetition allowed.

$$\text{No of 2's required} = \frac{19 - (2K+1)}{2} = 9-K$$

$$\text{No of ways to select} = \binom{(2K+2) + (9-K) - 1}{9-K} = \binom{10+K}{9-K}$$

$$\text{Total no of ways} = \sum_{K=0}^{9} \binom{10+K}{9-K} = \text{---}$$

(9) For "20"

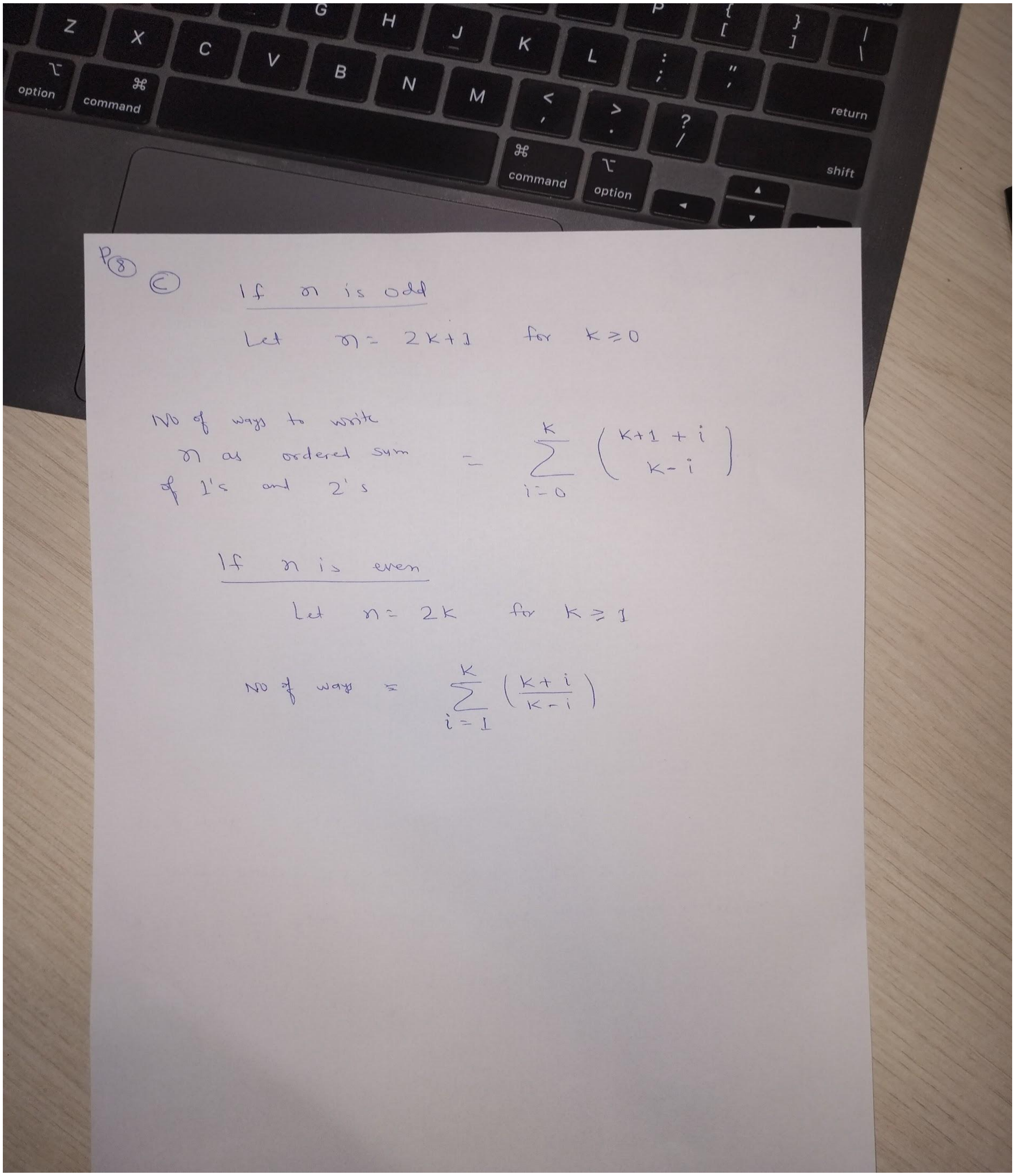
$$\text{No of 1's} = \text{even} = 2K \qquad 0 \le K \le 10$$

If ther are $2K$ 1's, there are $2K+1$ location with repetition allowed. No of 2's $= \frac{20-2K}{2} = 10-K$

$$\text{No of ways to select} = \binom{(2K+1) + (10-K) - 1}{10-K} = \binom{10+K}{10-K}$$

$$\text{Total no of ways} = \sum_{K=0}^{10} \binom{10+K}{10-K} = \text{----}$$

**Problem 8**



P8 (c)

If $n$ is odd

Let $n = 2k+1$ for $k \geq 0$

No of ways to write $n$ as ordered sum of 1's and 2's $= \sum_{i=0}^{k} \binom{k+1+i}{k-i}$

If $n$ is even

Let $n = 2k$ for $k \geq 1$

No of ways $= \sum_{i=1}^{k} \binom{k+i}{k-i}$

**Tutorial 1**

T 1

Let there be $x_i$ occurences of $i \in \{1, 2, 3, \dots r\}$

So we have,

$$x_1 + x_2 + x_3 + \dots x_r = n \quad \forall x_i \geq 0$$

Hence,

~~According to selecting~~

This matches with selecting $r$ objects with repeatation from $n$ objects s.t. we are selecting 0 or more of same object.

$$\Rightarrow \text{Total no of methods} = \binom{n+r-1}{n}$$

$$= \binom{n+r-1}{r-1} \quad \text{Ans}$$

# Tutorial 2

T (2) $\boxed{n \geq 2k}$

$0^*$ : Any no of 0's (0 or more)
$1^*$ : Any no of 1's (0 or more)

$$\underset{x_1}{0^*}\ \underset{x_2}{1^*}\ \underset{\underset{1^{st}}{2}}{10}\ \underset{x_3}{0^*}\ \underset{x_4}{1^*}\ \underset{\underset{2^{nd}}{2}}{10}\ \cdots\cdots\ \underset{x_{2k-1}}{0^*}\ \underset{x_{2k}}{1^*}\ \underset{\underset{k^{th}}{2}}{10}\ \underset{x_{2k+1}}{0^*}\ \underset{x_{2k+2}}{1^*}$$

(occurance of 10)

Here, we have exactly k occurrances of "10" and other strings of format above

(Because of $0^*\ 1^*$, form "10" is NOT being affected)

So, we can write this in form,

$$x_1 + x_2 + 2 + x_3 + x_4 + 2 + \cdots + x_{2k-1} + x_{2k} + 2 + x_{2k+1} + x_{2k+2} = n$$

$$\Rightarrow x_1 + x_2 + \cdots + x_{2k+2} + \underline{2 * k} = n$$

$$\Rightarrow x_1 + x_2 + \cdots x_{2k+2} = n - 2k.$$

Hence

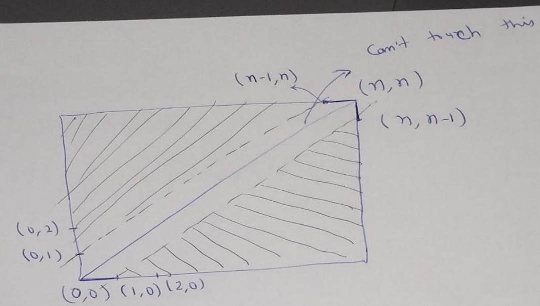
$$^{(n - \cancel{2k}) + (\cancel{2k}+2) - 1}C_{n-2k}$$

$$= \left( ^{n+1}C_{n-2k} \right)$$

$$= {}^{n+1}C_{(n+1)-(n-2k)} = \left( ^{n-1}C_{2k+1} \right) \text{ Ans}$$

**Tutorial 3 [Keeping the constraint -- at no point in time y > x]**

T 3




For the case, where robot could touch the diagonal (and NOT cross it),

the total path = Catlan no

= C(n)

Now, for this case,

If starting case is (0,0) ⟶ (1,0)

then final case would be (n, n-1) ⟶ (n,n)

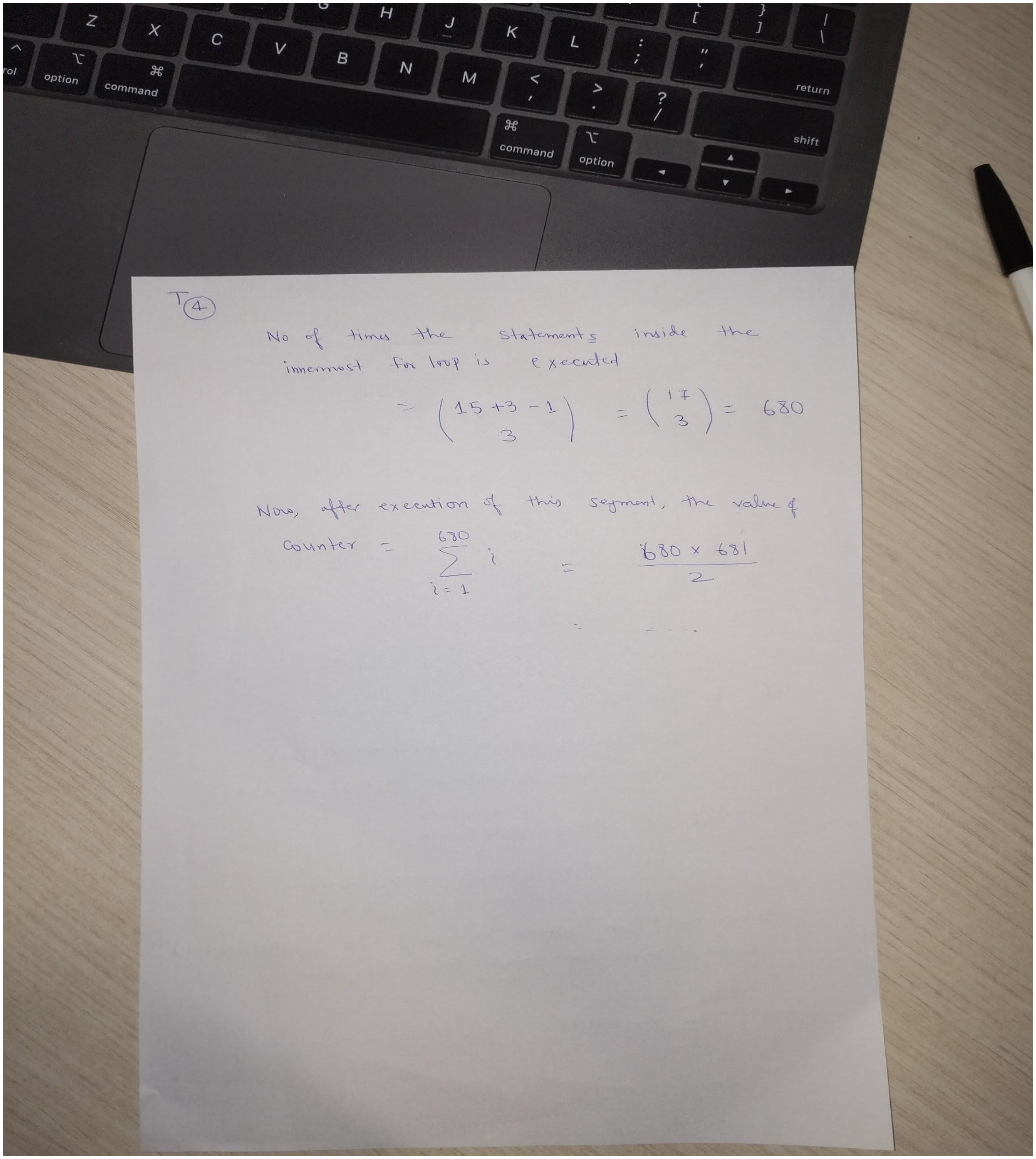
Similarly if starting (0,0) ⟶ (0,1)

then ending (n-1, n) ⟶ (n, n)

Hence we can assume it as a case of (n-1) x (n-1) where robot can touch the diagonal.

Hence, possible steps = C(n-1)

**Tutorial 4**



T 4

No of times the statements inside the innermost for loop is executed

$$= \binom{15+3-1}{3} = \binom{17}{3} = 680$$

Now, after execution of this segment, the value of

$$\text{counter} = \sum_{i=1}^{680} i = \frac{680 \times 681}{2}$$