

CS21201 Discrete Structures
Tutorial Problems
Elementary Counting Techniques

1. How many sorted arrays of size n are there if each element of the array is an integer in the range $1, 2, 3, \dots, r$?
2. How many **binary strings** of length n are there with exactly k occurrences of the **pattern 10**? (Assume that $n \geq 2k$)
3. In a grid measuring $m \times n$, an autonomous robot commences its journey from the origin $(0,0)$ with the goal of arriving at the destination point (m,n) . The robot exclusively maneuvers through the grid using rightward (R) and upward (U) movements. How many paths are possible with the constraint that the robot must refrain from touching the principal diagonal, except at the initial point $(0,0)$ and the terminal point (m,n) . (Assume $m=n$)
4. Let's recall the programming knowledge you've acquired in your PDS course and merge it with combinatorial techniques to re-evaluate the outcome of the subsequent program segment. Here, the variables $i, j,$ and k are integers, and your goal is to ascertain the final value of the integer variable '**counter**' after the execution of the given code snippet.

```
increment := 0 counter := 0
for i := 1 to 15 do
  for j := 1 to i do
    for k := 1 to j do
      increment := increment + 1
      counter := counter + increment
```