CS21201 Discrete Structures

Tutorial 3

Proof Techniques and Predicate Logic

1. Consider the following problem statements to be coded in first-order predicate logic.
   S1: There is a question which is answered by every candidate who answers at least one question
   S2: Every candidate answers some question
   G: Therefore, there is a question which is answered by all candidates

   Answer the following questions:
   a. List all predicates that you will use for encoding the problem
   b. Code S1, S2, and G using the predicates you defined
   c. Can you logically derive G from S1 and S2?

[Hint: Just like Universal Specification and Universal Generalization, you can also use Existential instantiation and Existential generalization. Thus, whenever ∃P(x) is true, you can instantiate x with a specific symbol b not appearing in the proof, and whenever P(b) is true for a specific b, you can generalize to ∃P(x).]

2. The game of Nim is played by two players, Alice and Bob. There are two piles with m and n sticks. The moves alternate between Alice and Bob. In each move, the player chooses one non-empty pile and removes one or more sticks from that pile. The player who fails to make the next move loses (that is, the player who makes the last move wins). Alice makes the first move. Prove the following assertions. [Just think of the strategy]
   a. If m = n, Bob can always win.
   b. If m ≠ n, Alice can always win

3. Using the principle of mathematical induction, prove the following statements
   a. For all $n >= 4$, the n-th Catalan number satisfies $C_n \leq 2^{2n-4}$
   b. The harmonic numbers $H_n = 1/1 + 1/2 + \dots + 1/n$ satisfy $H_n \leq ln(n) + 1 \; for \; all \; n \geq 1$.