CS21201 Discrete Structures
Tutorial Problems

Generating Function

1. A sequence $a_0, a_1, a_2, a_3,...$ is defined recursively as

   $a_0 = 1$

   $a_n = a_{n-1} + 2a_{n-2} + 3a_{n-3} + \cdots + na_0$ for $n \geq 1$.

   a. Derive a closed-form expression for the generating function A(x) of this sequence. Show all the steps of your derivation. (**Hint**: Use convolution.)

   b. From the generating function of Part (a), derive a closed-form formula for a_n. Show all the steps.

   c. From the formula of a_n derived in Part (b), deduce that $a_n = F_{2n}$ for all $n \geq 1$, where $F_0, F_1, F_2,...$ is the Fibonacci sequence. (**Hint**: Use the formula for Fibonacci numbers derived in the class.)

2. What is the generating function for the number of partitions of $n \in \mathbf{N}$ into summands that

   a. cannot occur more than five times

   b. cannot exceed 12 and cannot occur more than five times

3. Determine

   a. How many palindromic compositions of 11 start with 1? with 2? with 3? with 4?

   b. How many palindromic compositions of 12 start with 1? with 2? with 3? with 4?