

CS21201 Discrete Structures
Tutorial Problems

Set Sizes

1. Let A be an infinite set.
 - a. Prove that there is a map $A \rightarrow A$ which is injective but not surjective.
 - b. Prove that there is a map $A \rightarrow A$ which is surjective but not injective.

2. [Infinite Bit Sequences] As the name suggests, an infinite bit sequence is an infinite sequence of 0s and 1s. Denote S as the set of all infinite bit sequences. Let $\alpha(n)$ be the n th element of an infinite bit sequence $\alpha \in S$. Determine whether the following sets are countable or uncountable:
 - a. S
 - b. $T_1 = \{\alpha \in S \mid \alpha(n) = 1 \text{ and } \alpha(n+1) = 0 \text{ for some } n \geq 0\}$
 - c. $T_2 = \{\alpha \in S \mid \alpha(n) = 1 \text{ and } \alpha(n+1) = 0 \text{ for no } n \geq 0\}$