

CS21201 Discrete Structures
Tutorial 4

Induction (Generalized weak form, Strong)

1. Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two.
2. The game of Mini-nim is defined as follows: Some positive number of sticks are placed on the ground. Two players take turns removing one, two, or three sticks. The player who removes the last one loses. Use induction to show that:
The second player has a winning strategy if the number of sticks equals $4k + 1$ for some $k \in \mathbb{N}$; otherwise, the first player has a winning strategy.
3. Let n be a positive integer. Consider all non-empty subsets of $\{1, 2, 3, \dots, n\}$ that do not contain consecutive integers. Let S_n denote the sum of the squares of the products of the elements in these subsets.
For example, for $n = 3$, these subsets are: $\{1\}, \{2\}, \{3\}, \{1, 3\}$
Therefore S_3 is equal to $1^2 + 2^2 + 3^2 + (1 \times 3)^2 = 23$.
Prove that $S_n = (n+1)! - 1$ for all $n > 1$.
4. Given that every simple polygon with at least four sides has an interior diagonal, prove:
“A simple polygon with n sides, where n is an integer with $n \geq 3$, can be triangulated into $n - 2$ triangles.” using strong induction.