

CS21201 Discrete Structures
Practice Problems

Set Sizes

1. Prove that the set $A = \{a + ib \mid a, b \in \mathbb{Z}\}$ of Gaussian Integers is countable.
2. [Sets of Functions] We have the following sets, determine whether they are countable or uncountable.
 - a. [Submit] The set of all functions from \mathbb{N} to $\{1, 2\}$
 - b. The set of all functions from \mathbb{N} to \mathbb{N}
 - c. [Submit] The set of all functions from $\{1, 2\}$ to \mathbb{N}
 - d. The set of all non-increasing functions from \mathbb{N} to \mathbb{N} (Note: For a non-increasing function, $x < y \rightarrow f(x) \geq f(y)$)
3. Prove that the set of all permutations of \mathbb{N} is not countable.
4. Consider the set $S = \{a + b\sqrt{7} \mid a, b \in \mathbb{Z}\}$. Prove that $\mathbb{R} - S$ is uncountable.
5. Prove that the union of two sets equinumerous with \mathbb{R} is again equinumerous with \mathbb{R} .
6. Provide an explicit bijection between \mathbb{N} and $\mathbb{N} \times \mathbb{N}$. It should not be an exhaustive enumeration.
7. Determine whether the following sets are countable or uncountable
 - a. The set of all finite subsets of \mathbb{N}
 - b. The set of all infinite subsets of \mathbb{N}