CS21201 Discrete Structures
Practice Problems

Sets, Relations, Function (and some more)

1. Let A,B,C $\subseteq$ U. Prove that (A- B) $\subseteq$ C if and only if (A- C) $\subseteq$ B.

2. In how many ways can 15 laboratory assistants be assigned to work on one, two, or three different experiments so that each experiment has at least one person spending some time on it?

3. Professor Diane gave her chemistry class a test consisting of three questions. There were 21 students in her class, and every student answered at least one question. Five students did not answer the first question, seven failed to answer the second question, and six did not answer the third question. If nine students answered all three questions, how many answered exactly one question?

4. Let A, B$\subseteq$R, where A= $(x|x^2 - 7x < -12)$ and B= $(x | x^2 - x < 6)$. Determine A∪B and A∩ B.

5. **[Submit]** Let A, B, C, D be nonempty sets. [5.1.5]
   a. Prove that AXB $\subseteq$ CXD if and only if A$\subseteq$ C and B $\subseteq$ D.
   b. What happens to the result in part (a) if any of the sets A, B, C, D are empty?

6. Let A, B , C $\subseteq$ R^2 where A = $((x , y) | y = 2 x + 1)$, B = {(x, y)|y= 3x), and C= {(x, y)|x-y= 7). Determine each of the following:
   a. A ∩ B
   b. B ∩ C
   c. (A' U C')'
   d. B' U C'

7. Let a€R* where a>1. Prove that
   a. $\lfloor \lceil a \rceil / a \rfloor$=1; and
   b. $\lceil \lfloor a \rfloor / a \rceil$= 1
8. a) Find all real numbers x where $\lceil 3x \rceil = 3\lceil x \rceil$.

   b) Let n €Z* where n > 1. Determine all x€R such that $\lceil nx \rceil = n\lceil x \rceil$

9. **[Submit]** Let f:Z->N be defined by f(x) = 2x - 1 if x > 0 and f(x)= -2x, for x<0. Prove that f is one-to-one and onto.

10. Let A, B $\subseteq$ U. Prove that (A X B) ∩ (B X A) = (A ∩ B) X (A ∩ B)

11. Define a relation ρ on $A = Z \times N$ as $(a, b) \rho (c, d)$ if and only if $ad = bc$. Prove that ρ is an equivalence relation. Argue that A/ρ is essentially the set Q of rational numbers. In abstract algebra, we say that Q is the field of fractions of the integral domain Z. The equivalence class $[(a, b)]$ is conventionally denoted by a/b.

12. $S = \{(1,2),(2,1)\}$ is a binary relation on set $A = \{1,2,3\}$. Is it irreflexive? Add the minimum number of ordered pairs to S to make it an equivalence relation. Give the modified S.

13. In a ten-day period, Ms. Rosatone typed 84 letters to different clients. She typed 12 of these letters on the first day, seven on the second day, and three on the ninth day, and she finished the last eight on the tenth day. Show that for a period of three consecutive days, Ms. Rosatone typed at least 25 letters. Supp 13

14. Due to their outstanding academic records, Aditi and Rohit are the finalists for the outstanding mathematics students (in their college graduating class). A committee of 14 faculty members will each select one of the candidates to be the winner and place his or her choice (checked off on a ballot) into the ballot box. Suppose that Rohit receives nine votes and Aditi receives five. In how many ways can the ballots be selected, one at a time, from the ballot box so that there are always more votes in favor of Rohit?

15. Write each of the following arguments in symbolic form. Then, establish the validity of the argument or give a counter-example to show that it is invalid.

    a. If Shreya gets the supervisor's position and works hard, then she'll get a raise. If she gets the raise, then she'll buy a new car. She has not purchased a new car. Therefore either Shreya did not get the supervisor's position or she did not work hard.
    b. If there is a chance of rain or her red headband is missing, then Priti will not trim her lawn. Whenever the temperature is over 40°C. there is no chance for rain. Today the temperature is 45°C and Priti is wearing her red headband. Therefore (sometime today) Priti will trim her lawn.

16. Establish the argument's validity by Proof of Contradiction

$$[(p \rightarrow q) \wedge [(q \wedge r) \rightarrow s] \wedge r] \rightarrow (p \rightarrow s)$$

17. Among the 900 three-digit integers (from 100 to 999) those such as 131, 222, 303, 717, 848, and 969, where the integer is the same whether it is read from left to right or from right to left, are called palindromes. Without actually determining all of these three-digit palindromes, determine their sum.

18. If $n \in N$, prove that **$5 F_{n+2} = L_{n+4} - L_n$.**

19. Express the following in FOL. Use predicate when necessary. Avoid using predicate when a simple proposition would suffice.

    a. If tomorrow is a holiday, then some students of IITKGP will go home and the rest solve DM assignment.

    b. All students who study at IITKGP are residents of some hostel. However, some students are day scholars. Although, some students are day scholars, some hostel room is allotted by the hostel office

    c. Unless students sleep at 10 pm, they cannot get up early in the morning. There are students who sleep at 10 pm but still get up late. In each hostel, there is a student who either sleeps at 10 pm or does not sleep at all.