

CS21201 Discrete Structures
Practice Problems
Elementary Counting Techniques

1. For the word **ENGINEERING**, Determine.
 - a. Total possible arrangements
 - b. If all E's come together
 - c. If E's are never together
 - d. The arrangement must start with E and end with G
 - e. No E can be after N
 - f. In how many of the arrangements in part (a) are all the vowels adjacent?
2. In a grid measuring $m \times n$, an autonomous robot commences its journey from the origin (0,0) with the goal of arriving at the destination point (m,n). The robot exclusively maneuvers through the grid using rightward (R) and upward (U) movements. How many paths are possible with the constraint that the robot must **not** execute more upward (U) movements than rightward (R) movements at any juncture along its trajectory. (Assume $m \geq n$)
3. Consider a thought-provoking challenge that involves the enumeration of subsets. Given a set of natural numbers ranging from 1 to n, the task at hand is to determine the count of subsets of size k that possess a distinctive characteristic: there are more odd numbers than even numbers.
4. To raise money for a new Running Track, the General Secretary of Sports and Games, Gymkhana IIT KGP is sponsoring a race. Each participant pays a ₹20 entrance fee and has a chance to win one of the different-sized trophies that are to be awarded to the first five runners who finish.
 - a. If 50 people enter the race, in how many ways will it be possible to award the trophies?
 - b. If Krishna and Shyam are two participants in the race, in how many ways can the trophies be awarded to these two runners among the top three?
5. How many different positive integers n can we form using the digits 4, 5, 5, 6, 6, 7, 9 if we want to exceed 60,00,000?
6. ILLUMINATION, a hallmark tradition at IIT KGP, unfolds annually during the festive fervour of Diwali. This captivating ritual involves every residential hall, constructing intricate frameworks known as "chatai" using bamboo stripes. Over these structures, wire loops are deftly arranged to support traditional earthen lamps or diyas. The placement of these Diyas intricately weaves a thematic pattern, a reflection of the artistic vision of each student hall. Now, transitioning to the organizational realm, the task at hand revolves around the creation of a committee. Specifically, the challenge is to select 18 individuals

from a diverse pool of 15 men and 20 women. In how many ways can the selection be carried out if

- a. there are no restrictions?
 - b. there must be eight men and six women?
 - c. there must be an even number of men?
 - d. there must be more men than women?
 - e. there must be at least eight men?
7. You are given r red balls, g green balls, and b blue balls. Assume that r , g , and b are positive integers. Your task is to arrange the balls on a line subject to the following conditions. Find the count of all possible arrangements in each case.
- a. All blue balls appear together.
 - b. No two blue balls appear together.
 - c. No blue ball can appear after any green ball.
 - d. The arrangement must start with a blue ball and end with a non-blue ball.
8. Find the number of ways to write 19 as a sum of 1's and 2's if the order is relevant.
- a. Repeat the same for 20.
 - b. Generalize the results for n odd and for n even.
 - c. Repeat parts a and b if the order is **not** relevant.